Book reviews

Fourier Analysis, an introduction

E M Stein and R Shakarchi
Princeton University Press 2003
ISBN: 0-691-11384-X

This book is the first of a series of four, and was supported in part by the 250th Anniversary Fund of Princeton University (which accordingly predates Joseph Fourier's birth in 1768). The book with eight chapters and a helpful appendix on Riemann integration, is a good mixture of both older and more recent mathematics. After looking in Chapter 1 with the classical physical problems of vibrating strings and heat flow, Chapter 2 examines basic properties of Fourier Series and includes the concept of "good kernels" such as the Feyér kernel. Convergence of Fourier Series is further explored in Chapter 3, with applications (including some early 20th Century ones) in Chapter 4. Moving to the Fourier Transform on R, Chapter 5 introduces Laurant Schwartz's space of indefinitely differentiable rapidly decreasing functions, plus other 20th Century applications of Fourier Analysis of the Heisenberg uncertainty principles and the Black-Scholes equation from finance theory. Chapter 6 discusses the Fourier transform on R^d with applications to the Radon transform. Finite Fourier Analysis, including the Fast Fourier Transform and Fourier Analysis on Finite Abelian Groups is considered in Chapter 7. Chapter 8 on Dirichlet's Theorem (if q and l are relatively prime positive integers, then the arithmetic progression $l, l+q, l+2q, \ldots$ contain infinitely many prime numbers) is given as 'a striking application' of finite Fourier Series.

As above, Stein and Shakarchi's book is a good mixture of old and new mathematics. It is appealing for a breadth of scope with a clarity of exposition and is a useful reference. The book would also be helpful for honours students studying modern analysis. As such, it joins many significant books on Fourier Analysis, including the late Robert Edwards' impressive two volume work *Fourier Series: A modern introduction* (Holt, Rinehart and Winston 1967).

Philip Laird
School of Mathematics and Applied Statistics, University of Wollongong, Northfields Ave Wollongong NSW 2522
E-mail: phil_laird@uow.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Mathematical Puzzles A Connoisseur's Collection

Peter Winkler
A. K. Peters Natick 2004
ISBN 1-56881-201-9

Puzzles are disseminated among the mathematical literati in the same way that jokes are spread through the community — that is, by word of mouth. And like a good joke, the more appealing of these conundrums have been bandied around classrooms and offices, written on the backs of envelopes, and passed on from generation to generation until their origins have been lost in the mists of time. This book is a collection of more than one hundred of the best puzzles which have, to some extent, become a part of this mathematical folklore.

Of course, as with any compilation of this nature, there is a great deal of subjectivity as to what makes a mathematical puzzle worthy of inclusion. In the preface, the following five prerequisites are listed — amusement, universality, elegance, difficulty and solvability. But ultimately, with such a myriad of problems from around the world to choose from, the final say must come from the author. In this case the author is the reputable Peter Winkler, currently Director of Fundamental Mathematics Research at Bell Labs, a man who has had a distinguished career in both academia and industry. In some circles he is best known as the inventor of cryptologic methods for the game of bridge, currently illegal for tournament play in parts of the world. But most importantly, he is an avid puzzle solver and it is evident that this is indeed the collection of a connoisseur, as the title suggests.

This book is for anyone with a passion for mathematical puzzles; for those who love the thrill of being able to solve one as well as the frustration of being unable. There is no mention of concepts from higher mathematics such as groups or manifolds, and you can safely leave your calculus text on your bookshelf. Despite the lack of prerequisites, beware that the problems are generally very challenging. So apart from a certain amount of mathematical maturity, you will also require the determination to invest time in each problem before reading the solution. Remember that the satisfaction of solving a puzzle is proportional to its difficulty and also keep in mind the following words of the author.

"You can take pride in any puzzles you solve, and even more in any for which you find better solutions than mine."

So whether you are an amateur or a professional mathematician, a student or a teacher, this book will appeal to those with an appreciation of the beauty and ingenuity of mathematics. The following is one of the easier offerings from the book, which should give you a taste of the level of difficulty and flavour of the puzzles.

"Soldiers in the Field: An odd number of soldiers are stationed in the field, in such a way that all the pairwise distances are distinct. Each soldier is told to keep an eye on the nearest other soldier. Prove that at least one soldier is not being watched."

The puzzles are loosely classified into twelve mathematical areas, each of which constitutes a chapter of the book. The list of chapters includes numbers, combinatorics, probability, geometry and games. In turn, each of these chapters consists of about ten puzzles, followed by a section containing solutions and interesting comments for each of them. All of the solutions are concise, sometimes forsaking detail to make way for insight. The author takes the occasional excursion into the land of mathematical jargon, so for the less experienced reader, it may be necessary to skip the solution to a puzzle every now and then.

Unfortunately, the book is marred by a chapter entitled "Geography(!)", which consists of America-centric questions that require little insight, no mathematics, and access to an atlas of the world. The author claims that these are enjoyed by mathematical puzzle lovers, although I, and I am sure many others, do not fit this mould. However, in defense of this statement, and indicative of the humourous and personal style which pervades the book, the author writes

"My publisher has assured me that the book's price would be the same without this chapter, so it's free and you can skip it with a clear conscience."

For those who find the first ten chapters too basic, the author ups the tempo in the penultimate chapter of the book, entitled

"Toughies". And if this is not enough for you, the book concludes with a gem of a chapter dedicated to unsolved puzzles. Although the statements of these problems are tantalizing in their simplicity, many of them have earned notoriety for withstanding attempts to prove them from mathematicians of every calibre the world over.

As an enthusiastic solver and collector of mathematical puzzles myself, I was absolutely delighted with this book. The sheer density of mathematical ideas and challenges sets it apart from other books of a similar nature while the lucid yet informal style of exposition makes the journey from ignorance to enlightenment enjoyable. This book helped to remind me that beauty and ingenuity in mathematics need not be found in deep and abstract theories, but can be appreciated through puzzles which almost anyone can understand, though often few can solve.

Norman Do
Department of Mathematics and Statistics, The University of Melbourne, VIC 3010
E-mail: norm@ms.unimelb.edu.au

◇ ◇ ◇ ◇ ◇ ◇

17 Lectures on Fermat Numbers From Number Theory to Geometry

Michal Křížek, Florian Luca and Lawrence Somer
Springer Heidelberg 2001
ISBN 0-387-95332-9

This book was written in celebration of the 400th anniversary of the birth of Pierre de Fermat (1601 – 1665), to whom can be credited the first significant advances in number theory since classical times. There is a foreword (by Alena Šolcová) summarising his life and his other extensive and influential work, but the main focus of the book is on those numbers bearing his name, the Fermat numbers

$$F_m = 2^{2^m} + 1 \text{ for } m = 0, 1, 2, \ldots$$

The book, in its F_2 lectures, or chapters, is a comprehensive survey of the properties of the Fermat numbers, ranging from the well-known to the obscure, and containing some results due to the authors and not published elsewhere. It also mentions many connections with other areas of mathematics, and applications such as pseudorandom number generation and the use of the Fermat Number Transform in signal processing. Fermat numbers make their appearance in other surprising places, such as hashing schemes and the period-doubling bifurcations of the logistic equation.

Some of the material, such as the fact that no F_m has been found to be prime for $m > 4$, will be well-known to many. Fermat famously conjectured that all Fermat numbers are prime, but in 1732 Euler found that $F_5 = 641 \cdot 6700417$, and much work has been done since in finding factors of individual Fermat numbers or otherwise proving them composite. A survey of all such results known at the date of publication is included in the early chapters, and in tables in the appendix. Some of the properties are probably much less well known, such as the connection of Fermat numbers with Pascal's triangle. There are chapters on pseudoprimes and pseudosuperprimes, certain Diophantine equations, sums of reciprocals of Fermat numbers, and a list of open problems.

The famous theorem of Gauss that a regular polygon is constructible by ruler and compass if and only if the number of its sides is $n = 2^i p_1 p_2 \ldots p_j$, where $i \geq 0, j \geq 0, n \geq 3$ are integers and $p_1, p_2, \ldots, p_j$ are distinct prime Fermat numbers is mentioned in Chapter 4 (which is entitled The Most Beautiful Theorems on Fermat Numbers), and proved in Chapter 16, after a brief discussion of Galois theory. The final chapter describes in detail the construction of the regular 17-gon.

The authors state that the book is intended for a general mathematical audience, and that only basic results from algebra are assumed. The potential difficulties in trying to appeal to the wide spectrum from the amateur to the specialist have been recognized and successfully avoided. Chapter 2 consists of the necessary fundamental number theory, and though this could be skipped by the expert, there has been a particular emphasis on presenting new and unusual interpretations and proofs, often with a geometric flavour. Indeed, one of the very attractive features of the book is the liberal use of diagrams, graphs and photographs (71 illustrations in all). For the occasional proof requiring more specialized knowledge, the reader is referred elsewhere. After the first few chapters, the later sections can be read independently. There is an extensive list of references (17 pages) and a useful list of web sites.

The clear writing style and the evident enthusiasm of the authors make this book a pleasure to read or to dip into. It is not a text book, but might well inspire students at any level, and it succeeds in making what is potentially a dry and abstract topic fresh and full of connections. It would be a valuable addition to any library.

Alison Wolff
School of Pure Mathematics, University of Adelaide, Adelaide, SA 5005
E-mail: awolff@maths.adelaide.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Contemporary Abstract Algebra

Joseph A. Gallian
Houghton Mifflin 1998, 4th ed.
ISBN 0-395-86179-9

This is an exciting introductory text on abstract algebra! Before you decide that is an oxymoron, note Gallian's remarks: *We customarily assume that mathematics books, by the nature of the subject, must be humorless, lifeless, and sterile - and often they are. But I have broken this convention by including lines from popular songs, poems, quotations, biographies, historical notes, hundreds of figures, dozens of photographs, and numerous tables and charts.*

Gallian has broken the convention in other ways as well. He writes enthusiastically. He explains with great clarity. His illustrative examples are interesting, significant and memorable. His style is sufficiently informal that it is easy to read, yet sufficiently precise that it conveys the ideas accurately, economically, and with standard terminology. His style consistently reassures the reader that the author is an experienced guide and friend who communicates his subject as clearly and enthusiastically as possible. Witness an early tip he gives the reader: *The best way to grasp the meat of a theorem is to see what it says in specific cases.* And witness this remark by Paul Halmos that Gallian shares at the start of Chapter 2: *A good set of examples, as large as possible, is indispensable for a thorough understanding of any concept, and when I want to learn something new, I make it my first job to build one.*

To demonstrate these points, let me summarise Gallian's introduction to groups (pp.29–39). He begins with the symmetries of the square, described with characteristic clarity and imagination: *Suppose we remove a square from the plane, move it in some way, then put the square back into the space it originally occupied. Our goal in this chapter is to describe in some reasonable fashion all possible ways this can be done... To begin, we can think of the square as being transparent (glass, say), with the corners marked on one side with the colors blue, white, pink and green.* He soon produces the Cayley table for the group of symmetries of the square: *To facilitate future computations, we construct an operation table or Cayley table (so named in honor of the prolific English mathematician Arthur*

Cayley, who first produced them in 1854). The table follows, and he enthuses: *Notice how beautiful this table looks! This is no accident!* The example of the square introduces dihedral groups, soon illustrated by the logos of Chrysler and Mercedes-Benz, the symmetry of a pyramidal molecule such as ammonia, the beautiful rose windows in the Gothic cathedrals of Europe, some X-ray diffraction patterns of crystals, more logos, snowflake photographs, the 11-sided Canadian dollar coin, and the HIV virus. In just 11 pages Gallian introduces some key terminology and ideas about groups, and enriches the introduction by including several apt quotations, 19 illustrations of labelled squares accompanying the discussion of symmetries of the square, 39 further images, 23 carefully chosen exercises (with succinct "answers" to odd exercises in the Appendix), a list of references to 7 books on symmetry and a one-page biography of Abel (illustrated by a portrait, a stamp and a banknote), including the url of a website on Abel. The conceptual power of the subject is tapped into quickly (Ex. 8: *In D_n, explain geometrically why a rotation and a reflection taken together in either order must be a reflection.*) And the variety of illustrations conveys the relevance of the subject (Ex. 18: *Does the agitator of a washing machine have a cyclic symmetry group or a dihedral symmetry group?*).

Many chapters end with one-page biographies. That idea was pioneered well before this book, but Gallian's selection of biographical subjects is a clear indicator of his enthusiasm and originality. His subjects include a number of contemporary mathematicians, as well as significant figures of twentieth century mathematics. Look at the list of biographical subjects in the order in which they appear in the book: Abel, Sylvester, Cauchy, Cayley, Lagrange, Adleman, Galois, Jordan, Herstein, Jacobson, Dedekind, Noether, MacLane, Germaine, Wiles, Artin, Taussky-Todd, Kronecker, Kaplansky, Dickson, Sylow, Aschbacher, Gorenstein, Thompson, M. Hall, Escher, Pólya, Conway, Burnside, Hamilton, Erdös, Hamming, MacWilliams, Pless, P. Hall and Gauss. How much do you already know about the biography and mathematical significance of each of these? Gallian helps the reader gain an up-to-date view and appreciation of our cultural heritage, conveying at the same time a sense of the ongoing development and relevance of algebra.

Gallian's organization of subject matter is essentially standard, starting with assorted preliminaries (23 pages), followed by group theory up to the structure theorem for finite abelian groups (194 pages), then rings and integral domains (104 pages), fields (61 pages) and special topics (189 pages). The special topics are Sylow's Theorems, group generators and relations, symmetry groups, frieze groups and crystallographic groups, Burnside theory, Cayley digraphs, algebraic coding, Galois theory and cyclotomic field extensions. There are 1339 well-chosen exercises (averaging almost 40 per chapter), 239 supplementary exercises (after groups of related chapters), 35 computer exercises (averaging one per chapter), and an average of 4-5 references and selected readings (with brief descriptions) at the end of each chapter. The book would readily serve a two-semester sequence in abstract algebra, with selection from the special topics to adapt to time constraints. Since this is the fourth edition, presumably earlier editions established a positive reputation through classroom use. This edition certainly merits an enthusiastic readership and significant classroom use.

Roger B. Eggleton
School of Pure Mathematics, University of Adelaide, Adelaide, SA 5005, Australia
E-mail: roger@math.ilstu.edu

◇ ◇ ◇ ◇ ◇ ◇