



Book reviews

The Shoelace Book A Mathematical Guide to the Best (and Worst) Ways to Lace your Shoes

Burkard Polster
The American Mathematical Society
ISBN 0821839330

This little gem is precisely what its subtitle indicates. It provides analyses of how much string is required to tie shoelaces in various canonical patterns. These are mathematical, frictionless, inelastic shoelaces, and the analyses are combinatorial and geometric. That is, it is explicitly not a book of (topological) knot theory. To date only a handful of research papers on this topic have been published; this book constitutes a major statement of current knowledge.

The analyses are elegant, simple, and should be accessible to a reader with a basic understanding of calculus. The book has a formal mathematical layout, and is very readable. Beyond that, it must be mentioned that it is beautiful! Apart from many diagrams, it contains photos of real shoelaces and even cartoons relevant to the topic — recall Charlie Brown's problems. For good measure, it provides appendices containing cultural and historical notes.

This book is valuable in that it may motivate high school and undergraduate students to consider studying more mathematics for its own sake, as it illustrates that high school mathematics may be satisfactorily applied to practical design problems. I recommend it highly to school and public libraries.

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99 Points of Intersection

Hans Walser & Jean Pedersen
Mathematical Association of America
ISBN 0883855534

99 Points of Intersection is largely composed of a collection of 99 geometric figures where 3 or more lines or circles have a common point of intersection, with many of the figures having a certain aesthetic appeal. It has recently been translated from the original German, and is marketed at high school and undergraduate students.

The book is divided into three chapters. The first chapter serves as an introduction, and has some brief discussion of examples of intersection, ranging from concurrency of diagonals of a regular dodecagon, to interesting interference patterns generated by various families of functions.

The second chapter is presented largely visually, with a single page key at the beginning. Here 99 separate constructions are presented as a sequence of 3 diagrams outlining the construction process, with a fourth larger diagram for clarity. Each of these figures appear to have points of concurrency, the challenge is to demonstrate that these actually are points of concurrency. A motivated high school student could likely provide proofs for the first 11 diagrams, with the complexity of the problems increasing through the chapter.

The third chapter is a discussion of some general methods of proof, including software aided proofs, Ceva's theorem and invariance under various transformations. Some remarks on selected figures that appear in Chapter two are also provided.

Many readers will find sources of frustration in this book. The student is likely

to be frustrated by sections such as §1.1.2 and §1.1.3, page 4. In §1.1.2 a simple puzzle is presented using a regular dodecagon that can be solved by analysing angles and applying the law of sines. In §1.1.3 the following sentence appears:

Precisely, we can interpret the circumference of the dodecagon with the diagonals as chords as the Klein model of this geometry, and the same circle with the arcs orthogonal to it in its interior as the Poincaré model.

I fear that even the keen student may feel as though, in half a page, the book suddenly changed gears on them.

The more advanced reader may be frustrated by Section §1.3.3, where the first 30 Chebyshev polynomials are superimposed. Comment is made that various curves appear in the interference pattern generated, such as a parabola and a Lissajous curve, however the comment is limited to two brief paragraphs with no discussion on why such curves may appear.

The book has some interesting puzzles and does try to present some results in Euclidean geometry that the target audience would most likely not have had exposure to. My main criticism would be that in places the material is perhaps not self contained enough, particularly for high school students who would probably not have the means to chase down all the references given. The translators also note that the majority of the references are in German, however they have added some publications that are in English. The book would be best suited to a student with ready access to someone with a more advanced background, or perhaps as a reference for someone building a short interest course for high school or early undergraduate students.

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**Graph Algebras
CBMS regional conference
series in mathematics**

Iain Raeburn

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C^* -algebras arise in a wide variety of contexts. The present book, expanding on a set of lectures presented by the author in 2004, describes how they can arise from directed graphs. The first four chapters provide a general introduction to the theory, while the final six chapters cover more specialist aspects and generalizations.

The book is aimed at "a reader who has taken a first course in C^* -algebras, covering the Gelfand-Naimark Theorems, the continuous functional calculus, and positivity". Such is the influence of the group at Newcastle on operator theory in Australia, that most readers of the Gazette who meet this criterion are probably already familiar with the book. Indeed, given that over 50 of the articles on graph C^* -algebras from the bibliography are written by current or former Newcastle staff members, it may be that a majority of prospective Australian readers are already experts in the field. For this reason, my review will be aimed at those with no familiarity with C^* -algebras, to give them a very general introduction to the topic of the book.

Before this general introduction, I should point out that the author has clearly thought carefully about how to present the basic theory, using the simplest and clearest approaches. Then, in the final six specialist chapters, he has highlighted many interesting aspects of the field. Overall, the book is strongly recommended to all those with a basic background in C^* -algebras who want a general introduction to the theory of graph algebras.

Graph algebras arise from the geometry of inner product spaces. If K is a subspace of the complex n -dimensional Euclidean space $H = \mathbb{C}^n$ (or, more generally if

K is a closed subspace of a Hilbert space H), then every element h of H can be written uniquely as $h = h_1 + h_2$, where h_1 belongs to K and h_2 is orthogonal to it. The mapping P defined by $P(h) = h_1$ satisfies $P(h_1) = h_1$ for all h and therefore $P = P^2$. A simple calculation, using the orthogonality of h_1 and h_2 , shows that it also satisfies $P = P^*$, where P^* is the adjoint of P (determined by the property $\langle Ph, k \rangle = \langle h, P^*k \rangle$ for all h, k in H). P is called the *orthogonal projection* onto K ; its image is K and its kernel is the orthogonal complement of K .

If K_1 and K_2 are subspaces of $H = \mathbb{C}^n$ with the same dimension (or, more generally, are isometric closed subspaces of a Hilbert space H), then there is a linear map S mapping the orthogonal complement of K_1 to 0 and mapping K_1 isometrically onto K_2 . S is known as a *partial isometry*; it is characterized algebraically in terms of the orthogonal projections P_1, P_2 onto K_1, K_2 by the equations $S^*S = P_1$ and $SS^* = P_2$. P_1 is known as the *initial projection* of S and P_2 as its *range projection*.

Graph algebras are generated by families of partial isometries, with mutually orthogonal ranges, each associated with an edge of a directed graph. Infinite families are allowed, with some difficulties to overcome, but here I will just consider finite ones. So let G be a finite directed graph and, for each vertex v of G , let P_v be an orthogonal projection, with the property that each pair of vertex projections P_v, P_w satisfy $P_v P_w = 0$; this corresponds to the associated subspaces being orthogonal. For each (directed) edge e of G , with source vertex $s(e)$ and range vertex $r(e)$, let S_e be a partial isometry satisfying

$$S_e^* S_e = P_{s(e)}. \tag{CK1}$$

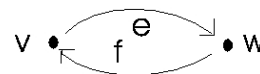
The graphical structure impacts on the algebra by means of the additional condition, applied whenever any edges arrive at v ,

$$P_v = \sum S_e S_e^*, \tag{CK2}$$

where the sum is taken over all the edges e with range $r(e) = v$. In recognition of

the writers of the fundamental paper underlying the theory of graph algebras, collections of partial isometries and orthogonal projections obeying the relations (CK1) and (CK2) for some directed graph G are known as Cuntz-Krieger families.

For example, the directed graph



has an associated Cuntz-Krieger family of 2×2 matrices: $S_e = e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, S_f = e_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, P_v = S_e^* S_e = e_{21} e_{12} = e_{22}$ and $P_w = S_f^* S_f = e_{11}$. Notice that (CK2) is satisfied because the only edge entering v is f , with $P_v = S_f S_f^*$, and the only edge entering w is e , with $P_w = S_e S_e^*$. The Cuntz-Krieger family contains each of the matrix units $e_{11}, e_{12}, e_{21}, e_{22}$, so generates the $*$ -algebra of all two by two matrices.

The directed graph above is also associated with the following Cuntz-Krieger family, consisting of pairs of 2×2 matrices: $S_e = (e_{12}, e_{12})$ and $S_f = (e_{21}, -e_{21})$, with $P_v = (e_{22}, e_{22})$ and $P_w = (e_{11}, e_{11})$. The $*$ -algebra generated by this family is not isomorphic to the algebra $M_2(\mathbb{C})$ of all 2×2 matrices, but is instead the direct sum of two copies of $M_2(\mathbb{C})$: for example the element $(e_{22}, 0)$ can be realized as $\frac{1}{2}(S_f S_f^* + S_e S_e^*)$.

The *graph algebra* A associated with a directed graph is not only generated by a Cuntz-Krieger family for the graph, but also has a universal property: every other C^* -algebra generated by a Cuntz-Krieger family for the graph is a quotient of A . For the graph above, constructing A involves moving outside the finite dimensional world and taking the norm closure to produce the C^* -algebra $A = C(S^1, M_2(\mathbb{C}))$ of continuous 2×2 matrix-valued functions on the unit circle (as is explained in the book). As required by the universal property, both the two algebras $M_2(\mathbb{C})$ and $M_2(\mathbb{C}) \oplus M_2(\mathbb{C})$

above are homomorphic images of A , obtained by evaluating at a single point or at a pair of points.

Other directed graphs, such as a single vertex with n loops, require all associated Cuntz-Krieger families to be infinite-dimensional. For this example, all Cuntz-Krieger families generate isomorphic C^* -algebras (known as O_n) and these are simple, i.e. have no non-trivial norm closed ideals. The reasons for these facts, together with many other fascinating results linking the structure of the graphs to those of the algebras, can be found in the book under review.

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103 Trigonometry Problems: From the Training of the USA IMO Team

Titu Andreescu and Zuming Feng
 Birkhäuser Boston 2005
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103 Trigonometry Problems is the third of its nature by the two Authors. The first two being *101 Problems in Algebra* and *102 Combinatorial Problems*. All three books contain instructive problems and techniques used in the training and testing of the USA International Mathematical Olympiad (IMO) team.

As such the book is appropriate for any highly capable secondary school student. Of course it is suitable for any person who is interested in problem solving after the style of the IMO. As it turns out most Olympiad type mathematics problems are not trigonometric in nature, yet many can be cracked by using trigonometry in clever ways.

The book is divided into five sections and a glossary. The first being on trigonometric fundamentals, the next two sections containing the 103 problems and the last two sections containing solutions to those problems.

The first section gives the basic definitions, properties and proofs of the trigonometric functions. This includes the addition and subtraction formulas, the sum-to-product and difference-to-product formulas, the extended sine rule and the cosine rule. These are applied to derive the theorems of Ptolemy, Ceva, Menelaus, Stewart, Heron, Brahmagupta and De Moivre. There are a few worked examples of applications to problems in geometry and algebra, in particular inequalities. In a number of instances diversions outside of trigonometry are discussed. For example, the section “Think Outside the Box”, shows how by considering a dilation one is led to the construction, using only straight edge and compass, of a square inscribed inside a triangle. There is a section on vectors leading up to the Cauchy-Schwartz inequality which is interpreted as $\|u\|\|v\|/|u \cdot v| = |\cos \theta| \leq 1$. A section on complex numbers connects to De Moivre’s formula. The important limit $\lim_{\theta \rightarrow 0} (\sin \theta)/\theta$ is discussed. Although this section gets off the ground quite quickly, there are some parts that are treated perhaps a little too carefully. For example, because the proofs of the addition formulas are appropriately geometric in nature, there is much paying of attention to noting that their proofs only apply to restricted intervals in the beginning. Yet this over attention to detail gets overlooked on page 9 by the use of the subtraction formula before it is developed.

The second section contains 52 introductory problems. The third section contains 51 advanced problems. Few of the problems are very geometric, in fact most are algebraic in nature. More than half of the advanced problems are inequalities. Nonetheless a wide array of problems that can be

tackled with a geometric approach are to be found. The fourth section contains solutions to the introductory problems. The fifth section contains solutions to the advanced problems. In both instances multiple solutions are often offered. The glossary contains supplementary material needed in the solutions to some of the problems.

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Research Problems in Discrete Geometry

Peter Brass, William Moser and János Pach
 Springer
 ISBN 0-387-23815-8

When asked to review this book, my first thought was ‘What is discrete geometry?’ In attempting to answer this question I can do no better than quote the book’s preface by Paul Erdős:

As a matter of fact, I cannot even give a reasonable definition of the subject. Perhaps it is not inappropriate to recall the following old anecdote. Some years ago, when pornography was still illegal in America, a judge was asked to define pornography. He answered: “I cannot do this, but I sure can recognize it when I see it.”

After reading the book I too can recognize discrete geometry when I see it.

The book has its origins in a list of 14 problems originally proposed by Leo Moser and distributed by William Moser under the same title in 1977. At least seven further editions of problems have appeared since in various forms, culminating in the book under review.

This book contains a wealth of problems and conjectures divided into 11 chapters, with each chapter divided into many sections. Each section outlines the history of

the problems included, details any progress made and has its own extensive bibliography.

The authors begin with problems concerning packings and coverings and there are three chapters of such problems. A packing is a collection of subsets of a domain such that no two subsets have an interior point in common, while a covering is a collection of subsets such that every point of the domain is contained in at least one subset. Restrictions may be placed on the type of subset used, for example spheres, or on the domain. A common theme is to determine the optimal density of the packing/covering and which configurations achieve this. The most famous problem in this area is Kepler’s Conjecture that the maximum density of a packing of equal balls in \mathbb{R}^3 is $\pi/\sqrt{18}$. On the recent highly computational proof of this conjecture the authors state that ‘so far no one has found any serious gap in the approach of Hales and Ferguson, although no one has been able to fully verify it either.’ There is also a chapter on tilings, that is, a collection of subsets which is both a packing and a covering.

Another fertile problem area arises from configurations of points. Some of the main themes are: What is the maximum number of occurrences of the same distance between n points? The minimum number of distinct distances? The minimum number of distinct directions determined by the points? The minimum number of lines passing through precisely two of the points? There are also chapters on graph drawings, lattice point problems, and geometric inequalities.

All in all, the book under review is very comprehensive, almost encyclopedic in its treatment, and will make a very good reference, especially for those starting out in the field and looking for problems to work on.

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