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# The Australian Mathematical Society

## Gazette

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The *Gazette* seeks to publish items of the following types:

- Mathematical articles of general interest, particularly historical and survey articles
- Reviews of books, particularly by Australian authors, or books of wide interest
- Classroom notes on presenting mathematics in an elegant way
- Items relevant to mathematics education
- Letters on relevant topical issues
- Information on conferences, particularly those held in Australasia and the region
- Information on recent major mathematical achievements
- Reports on the business and activities of the Society
- Staff changes and visitors in mathematics departments
- News of members of the Australian Mathematical Society

Local correspondents are asked to submit news items and act as local Society representatives. Material for publication and editorial correspondence should be submitted to the editors.

### Notes for contributors

Please send contributions to [gazette@austms.org.au](mailto:gazette@austms.org.au). Submissions should be fairly short, easy to read and of interest to a wide range of readers. Technical articles are refereed.

We encourage authors to typeset technical articles using  $\text{\LaTeX} 2_{\epsilon}$ ,  $\text{\AMS-L\TeX}$  or variants. In exceptional cases other formats may be accepted.

We would prefer that other contributions also be typeset using  $\text{\LaTeX} 2_{\epsilon}$  or variants, but these may be submitted in other editable electronic formats such as plain text or Word.

We ask that your  $\text{\TeX}$  files contain a minimum of definitions, because they can cause conflict with our style files. If you find such definitions convenient, please use a text editor to reinstate the standard commands before sending your submission.

Please supply figures individually as postscript (.ps) or encapsulated postscript (.eps) files.

Deadlines for submissions to Volumes 34(5) and 35(1) of the *Gazette* are 1 October 2007 and 1 February 2008.

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# Editorial

Welcome to the fourth issue of the *Gazette* in 2007.

We've got some good news from one of the editors — Rachel Thomas is expecting her second child in October, and has now commenced maternity leave until mid-next year. All the best to you, Rachel, and we hope to see you back at the *Gazette* next year!

In this issue, Michael Cowling announces the Society's move to publish *The ANZIAM Journal*, the *Bulletin of the Australian Mathematical Society* and the *Journal of the Australian Mathematical Society* through Cambridge University Press, and the implementation of an electronic manuscript handling system. This system will also handle *Gazette* submissions in the near future; more details will be available from the *Gazette* website soon.

We are pleased to present the second Classroom Notes column of the year. Terry Mills writes about a course on the history of mathematics, which he has been delivering to undergraduate students at La Trobe University since 2005. This course is based on historical documents, rather than modern text books, and covers the history of elementary mathematics from Ancient Egypt and Greece as well as Medieval Europe.

If you think your teaching practice is novel and of interest to others, we would like to hear about it for the Classroom Notes. The aim of this column is to share experiences and successful approaches in teaching mathematics.

The winner of Norman Do's Puzzle Corner 2 is Natalie Aisbett from RMIT, who has won a \$50 bookshop voucher. Congratulations! As in the last issue, some of the submitted solutions to Puzzle Corner 2 can be found right after this issue's Puzzle Corner. Please submit your solutions to Puzzle Corner 4 by 1 November for a chance to win a bookshop voucher.

We have a report on the Mathematics and Statistics in Industry Study Group (MISG2007), held at the University of Wollongong in February this year, and another report on the ICIAM 2007 meeting in Zurich. A simple approach from 'Rolle to Cauchy' was written by J. Koliha and one of his students, Peng Zhang. What makes this paper special is that Peng is a first-year student, whose intelligent enquiry led to the simple proof of Cauchy's mean value theorem!

We are also inviting contributions for the Maths@Work column. If you know of someone who may have an exciting story to tell about how they use their mathematical training and skills in their job, please encourage them to contact us so we can publish their story.

Happy reading.

Birgit and Eileen



# President's column

**Peter Hall\***

## AMSI

Many of you will be aware that, following last December's launch of the National Strategic Review of Mathematical Sciences Research, a number of members of the Society held discussions with public servants, politicians, and scientists from other disciplines. The aim was to deliver the message of the review. The Australian Mathematical Sciences Institute played a crucial role in this process.

In part, that role was immediate and direct. For example AMSI staff, especially Phil Broadbridge, Garth Gaudry and Jan Thomas, helped organise, and were closely engaged in, the dialogue that led up to the forum on the review in February and to the federal budget three months later. These activities will, I am sure, continue into the future.

However, in a broader and less explicit way, AMSI played an even more important role as a vehicle for the mathematical sciences. AMSI provides us all, whether we work in a private company, a university, a government agency, or wherever, with a connection to just about everything that touches mathematics in Australia. These vital linkages are visible and tangible throughout the range of activities in which AMSI is involved, from writing school textbooks to running high-level research workshops and engaging in mathematical work for industry. Many members of the Society benefit directly from AMSI's enterprise, but it's the indirect benefit that concerns me most here.

The breadth of AMSI's work is also apparent through the Institute's membership, which encompasses many institutions willing to pay for the privilege of being engaged in the mathematical sciences right across their spectrum in Australia. The size and scope of AMSI's paid-up membership and programs, as much as anything else, express AMSI's reach and commitment impressively well to those outside the profession. In order to make the point to you, inside the profession — and to make clear to you the substantial indirect benefits that we gain from AMSI — let me briefly describe one of the meetings held in Canberra early this year, after the launch of the review but before the forum.

The aim at the meeting was to convey information about the review to a high level in the public service and among politicians. We were talking to a senior person who was, I felt sure, both sympathetic and sceptical. On the latter side he was concerned that the review report might be presenting only part of the picture of the mathematical sciences in Australia — the part that represents those of us who work in universities. The government wasn't really interested in increasing its investment in university teaching and research, not unless it could see the whole mathematics picture, ranging from school mathematics teaching

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through to universities, business and industry; and unless it was convinced that the mathematical sciences community was unified and communicating within itself and with the world outside. Most importantly, we had to be totally committed to helping ourselves before the government would raise a hand to assist us.

The role played by AMSI was essential to substantiating the sweeping scope of mathematics, to providing the vital linkages between university mathematics and the rest of the world, and to describing our commitment to work for the good of mathematics as a whole. Without AMSI we would have foundered. The review report would have struggled to get traction in the federal sphere, and I feel sure there would not have been the extra funding for mathematics and statistics that we received in the federal budget. In the view of the person to whom we spoke in Canberra, AMSI supplied the glue that connected individual mathematical scientists to other professionals in different areas of the economy and in other parts of the nation, and in particular which linked university mathematics to schools, business, industry and government.

Ironically, AMSI is still not financially supported by the federal government as a mathematical sciences institute, although the government acknowledges AMSI's achievements and merits. From our point of view, as working mathematicians in a wide variety of fields, we should appreciate what AMSI has done for us in the past, both directly and indirectly, and what it can do in the years ahead. We must continue to support AMSI, indeed increase our support, because of the many opportunities AMSI brings us and the invaluable connections it provides.

Of course, this is not to say that AMSI should be immutable. Particularly when we can ensure a more reliable line of funding, AMSI should evolve so as to give ever better service, and still more opportunities, to the profession. Right now, however, AMSI needs our heightened support as it seeks funding that will take it more securely into the future.



Peter Hall is a statistician, with interests in a variety of areas of science and technology (particularly the physical sciences and engineering). He got his first degree from The University of Sydney in 1974, his MSc from The Australian National University in 1976, and his DPhil from University of Oxford in the same year. Peter is interested in a wide variety of things, from current affairs to railways and cats.



# Classroom notes

## A Course on the History of Mathematics

T.M. Mills\*

### Introduction

Since 2005, I have had the opportunity to teach a subject named *History of Mathematics* to undergraduate students at La Trobe University. In 2007, my colleague Grant Cairns and I taught the subject together. A key feature of the subject is that it is based on historical documents rather than a modern text book. The purpose of this paper is to share our experiences in presenting the subject. The terms ‘subject’ and ‘course’ are used interchangeably below.

### Context

This section describes the context in which the subject was delivered.

### Aims of subject

The aims of the subject are described to the students as follows.

Mathematics is created by human beings and hence is connected with the culture, the times, and the place in which this creative activity takes place. Thus, mathematics has a history — an interesting history. In *History of Mathematics*, we will focus on mathematics up until the middle ages. We will

- reflect on mathematical ideas that we met in primary school;
- encounter some new mathematical ideas;
- expand our interests in history;
- study some classic mathematical writings;
- see mathematics as an intellectual endeavour;
- gain experience in research, problem solving, and communication.

If you aspire to a teaching career, then the subject will be particularly useful in your professional training.

### Pre-requisites

There are no pre-requisite subjects for *History of Mathematics*. We assume only that the students are acquainted with the mathematical ideas encountered at school, namely arithmetic, basic algebra, and some geometry. In particular we do not as-

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sume that the students have ever studied calculus. The subject is an elective subject and is not compulsory in any degree program.

Consequently, the subject is limited to the history of elementary mathematics. We take this limitation even further, and restrict the subject to the history of ancient, elementary mathematics. Such a course ignores the great advances in mathematics since Newton. On the other hand, there is plenty of interesting material to study even with this restriction, especially in a one semester course. It is easier to appreciate the history of a piece of mathematics if one is confident in one's own knowledge of the mathematics itself.

In 2007, the subject was offered to students on the Bundoora and Bendigo campuses of La Trobe University. The students enrolled in the subject came from a range of degree programs. The Bundoora students were enrolled in the Bachelor of Science degrees, although only a couple of them were majoring in the mathematical sciences. Almost all the Bendigo students were undertaking a Bachelor of Education with the intention of becoming mathematics teachers, mainly at the primary or lower secondary levels.

## Classes

The course consists of four classes each week for 13 weeks. With small classes, there is no need to distinguish between lectures and tutorials. About one hour each week is devoted to watching a DVD. The DVD documentaries give students a flavour of the culture and times in which the mathematics was created. The DVDs were readily obtained and copies were placed in the library.

## Structure, content and resources

*History of Mathematics* is divided into three approximately equal parts. Each section is based around an original mathematical document (in translation).

### Ancient Egypt

The first part of the course deals with the mathematics of ancient Egypt. Our main source of information about the mathematics of ancient Egypt is the *Rhind Mathematical Papyrus* (RMP), which is now housed in the British Museum. Fortunately, the Heyward Library at La Trobe University Bendigo has a copy in [2]. I obtained a copy of the work by the egyptologist Peet [13] through a rare-book dealer. Peet's work is a copy of a facsimile edition of RMP, a translation, and a commentary. Supplementary material was found in [1], [6], [14]. Background material on ancient Egypt was provided in the documentaries [4].

In many text books, the mathematics of ancient Egypt is covered in only a few pages. However, one can study the mathematics in ancient Egypt in some depth by reading RMP for four weeks.

In addition, students learn elementary Egyptian in the process of studying this manuscript. Fortunately, the Egyptian and the mathematics in RMP are fairly simple; the book by Allen [1, Chapters 1 and 9] is an excellent source. Learning from materials in a language other than English is a unique feature of this subject.



## Ancient Greece

The second part of the course deals with the mathematics of ancient Greece. We read Euclid's *Elements*. We use Heath's translation in [7], and [9]. We cover Book 1, Propositions 1–13 (approximately) of the *Elements*.

The style of this section is good old-fashioned 'theorem and proof' mathematics. Students encounter definitions, axioms, theorems, proofs — and the limitations of Euclid's proofs. Here, students have more exposure to theorems and proofs than in many first year university subjects in mathematics. Students find the material a little dry initially, yet, by the end of this section, they became more critical of the proofs. We hope that they gained admiration for this magnificent intellectual achievement.

There are some flaws in Euclid's proofs. Herein lies another advantage of studying original documents. An error in a modern text book is irritating, whereas finding an error in Euclid is exciting.

Background material on ancient Greece was provided in the documentaries [5].

## Medieval Europe

The third section of the course deals with the mathematics of medieval Europe. We read Fibonacci's 13th century work *Liber Abaci* [17]. Fibonacci's aim is to introduce Italians to the Hindu-Arabic numeral system and how it is used in arithmetic. He is hampered by the lack of symbols  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $=$  and hence the writing is rather turgid. During four weeks we cover Chapters 1–5. In reading Fibonacci, we encounter his ways of presenting arithmetic and its applications. We find him using 'casting out nines' as a means of checking arithmetical calculations. He introduces us to lattice multiplication that is so often used in schools in the 21st century [17, Chapter 3]. We hope that by the end of this section, students will appreciate the important role that Fibonacci played in the history of mathematics.

Reading Fibonacci was supplemented by the four German documentary programs [3]. The second program in this series highlights the influence of Arab scholars in Spain on science in the middle ages.

## Assessment

There are two types of assessment: assignments and an examination.

### Assignments

There are two assignments. Each assignment is a 1000-word essay and contributes up to 25% to the final mark in the subject.

The topic of the first essay is 'Who's who in ancient Greek mathematics'. This allows the students the opportunity to supplement their study of Euclid and read about other mathematicians of the period. They may write short paragraphs about many people as in the traditional 'Who's who' format. Alternatively, they may choose to concentrate on just two mathematicians.

The second essay may be on any topic provided that the student discusses it with the lecturers before they embark on the project. At first, students found it difficult to choose a topic and needed guidance. However, they responded very well. They chose topics such as women in mathematics, the history of Sudoku, the origins of algebra, Roman numerals, history of zero, and a description of *The Nine Chapters on the Mathematical Art* [15]. One student gave a 20-minute presentation rather than write an essay. A student who was majoring in mathematics used the opportunity to explore aspects of the Newton–Leibnitz dispute.

Each assignment has a weighting of 25% towards the final mark. Students were given a copy of the marking scheme at the beginning of the semester to encourage certain characteristics in their writing. The marking scheme is in Table 1.

**Table 1.** Scheme for marking essays

Characteristic	Marks
Is the aim of the essay clearly stated?	2
Is the essay well structured?	3
Is the argument convincing?	4
Is the text linked to the sources?	2
Is there evidence of <b>your</b> ideas in the essay?	2
Are sources referenced correctly?	3
Is there variety in types of sources (books, articles, web sites)?	1
Is the language clear?	2
Have you demonstrated some capacity for mathematical word-processing?	1
Is the length of the essay satisfactory?	1
Is the presentation satisfactory?	1
Is the spelling satisfactory?	1
Is the grammar satisfactory?	1
Is punctuation satisfactory?	1
Total	25

Students are asked to demonstrate some capacity for mathematical word-processing because I suspect that, in their university training, students get little experience in this new art. Students who aim to become mathematics teachers will have to set tests and prepare assignments or notes for their students. Students who go on to industry may have to write technical reports. As readers of the *Gazette* know only too well, technical word-processing is non-trivial.

Since prevention is better than cure, I offered to comment on draft copies of the essays, in the second essay in 2007. When reading a draft copy, I tend to give more detailed advice on the work than if I were marking a final paper because I am trying to help the student to improve *this* essay rather than future essays.

## Examination

The final examination is a two-hour, closed book examination with no calculator. It has a weighting of 50% towards the final mark in the subject. Below is a sample of questions from the examinations in 2006 and 2007. Attached to the examination was a list of definitions, common notions and the statements of Propositions 1–13 from Euclid's *Elements*, Book 1.

- (1) **Ancient numeral systems:**  
Construct a multiplication table from  $1 \times 1$  up to  $5 \times 5$  using:
  - (a) the numeral system of ancient Egypt,
  - (b) the numeral system of ancient Greece, and
  - (c) the numeral system of ancient Rome.
- (2) **Ancient Egypt:**
  - (a) How would you multiply by 10 in ancient Egypt?
  - (b) Use an example to describe how one would multiply any two natural numbers in ancient Egypt.
  - (c) What mathematical principle underpins this method of multiplication?
- (3) **Teaching students about numeral systems:**  
Describe how you would introduce primary school students to Roman numerals.
- (4) **Euclid:**  
Using Euclid's approach, prove any two of the first 10 of Euclid's propositions on the attached list according to Euclid's methods.
- (5) **Euclid:**
  - (a) Explain the meaning of the Latin term 'reductio ad absurdum'.
  - (b) Show how Euclid used this method to prove Proposition 6 on the attached list.
- (6) **Euclid:**  
For Proposition 5 of Book 1 of the Elements, give Euclid's proof in detail. In giving the proof, try to avoid using notions that were not employed by Euclid.
- (7) **Medieval multiplication:**
  - (a) Describe the method used by Fibonacci in Chapter 2 of *Liber Abaci* to carry out the multiplication  $37 \times 49$ .
  - (b) Describe and explain his method for checking the result.
- (8) **Medieval division:**
  - (a) Describe the method used by Fibonacci in Chapter 5 of *Liber Abaci* to carry out the division of 1346 by 4.
  - (b) Describe the method used by Fibonacci in Chapter 5 of *Liber Abaci* to carry out the division of 18456 by 17.
  - (c) Briefly, compare Fibonacci's methods with how you might carry out these calculations by hand.

## Conclusions

Reading about Euclid and reading the *Elements* are totally different experiences. Approaching the history of mathematics through original materials provides us with intimate connections to the author, and the times and the place in which the mathematical ideas were expressed. This is the principal advantage of the approach. Laubenbacher and Pengelley [11], [12] describe their experience in using this approach for presenting the history of mathematics to students at various levels.

Furthermore, there are additional advantages.

The subject allows students to develop 'graduate attributes' in ways that other mathematical subjects do not. For example, library research, essays, class presentations, and team projects may be incorporated into *History of Mathematics*.

The subject encouraged students to develop their own opinions much more than in other sections of the university mathematics curriculum.

Indeed, in this subject, students with a relatively strong background in mathematics were not especially advantaged. *History of Mathematics* offered so much scope for exploring different aspects of mathematics, that students who are not particularly strong in mathematics could use other skills effectively. Thus, the subject can broaden the appeal of our discipline.

Although the curriculum was restricted to the history of ancient, elementary mathematics, there is a great deal of interesting material available for study. Furthermore, the history of *elementary* mathematics has the potential to be enjoyed by more students than the history of *advanced* mathematics.

One could vary the course by using other original mathematical documents. The works by Archimedes, Appolonius of Perga, and Nicomachus are available in [8]; note that this version of Archimedes' work is a modernised version of the works of Archimedes rather than a faithful translation. The first volume of a faithful translation of the works of Archimedes has been produced by Netz [10]. A section on the mathematics of China could be based on the classic work [15]. Fibonacci's *Book of Squares* [16] would provide an excellent introduction to number theory.

We often hear the argument in universities that mathematicians should teach mathematics subjects. Who should teach *History of Mathematics* — a mathematician or a historian? By studying original mathematical manuscripts, the subject has a very strong mathematical focus and hence, with this approach, it is reasonable, that the subject be taught by a mathematician.

The documentaries on the DVDs gave students a feel for the place and times in which the mathematics was created. These programs linked our subject with other disciplines such as archaeology, ancient and medieval history, and the history of science, technology, architecture, building, and agriculture.

The students did learn some *mathematics* in this subject. They did not know about Egyptian fractions before undertaking this subject. Some had encountered Fibonacci only in connection with Fibonacci numbers. Most students knew nothing about Euclid at the beginning of the course, and had little prior experience in proving theorems until they immersed themselves in the *Elements*. None knew about casting out nines before the course.

Furthermore, students developed a clearer appreciation of things that we take for granted in mathematics. For example, Fibonacci shows us the importance of place value in our number system.

More importantly, *History of Mathematics* provides new and different ways to gain confidence in mathematics, and is a bridge between 'the two cultures'.

### Acknowledgements

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# Puzzle corner 4

**Norman Do\***

Welcome to the Australian Mathematical Society *Gazette's* Puzzle Corner. Each issue will include a handful of entertaining puzzles for adventurous readers to try. The puzzles cover a range of difficulties, come from a variety of topics, and require a minimum of mathematical prerequisites to be solved. And should you happen to be ingenious enough to solve one of them, then the first thing you should do is send your solution to us.

In each Puzzle Corner, the reader with the best submission will receive a book voucher to the value of \$50, not to mention fame, glory and unlimited bragging rights! Entries are judged on the following criteria, in decreasing order of importance: accuracy, elegance, difficulty, and the number of correct solutions submitted. Please note that the judge's decision — that is, my decision — is absolutely final. Please e-mail solutions to [N.Do@ms.unimelb.edu.au](mailto:N.Do@ms.unimelb.edu.au) or send paper entries to: Gazette of the AustMS, Birgit Loch, Department of Mathematics and Computing, University of Southern Queensland, Toowoomba, Qld 4350, Australia.

The deadline for submission of solutions for Puzzle Corner 4 is 1 November 2007. The solutions to Puzzle Corner 4 will appear in Puzzle Corner 6 in the March 2008 issue of the *Gazette*.

I would like to thank Jamie Simpson of Curtin University for sending me some interesting problems, one of which appears as the first entry in this Puzzle Corner. If you have your own favourite problems which you would like to share, then I would most certainly welcome your contribution. Problems may be submitted by the same means as solutions, as described above.

## Digital sequences

The digits from 0 to 9 are written in a row such that each digit other than the leftmost is within one of some digit to the left of it. In how many ways can this be done?

## Blindfold balance



Thirty coins lie on a table with precisely seventeen of them showing heads. Your task, should you choose to accept it, is to form two groups of coins, not necessarily of the same size, such that each contains the same number of coins showing heads. Unfortunately, you happen to be blindfolded and cannot feel the difference between the two sides of a coin. How can you perform the task?

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### Chicken O’Nuggets

A certain Irish fast food chain sells Chicken O’Nuggets in large, humongous or colossal packs. A large pack contains 6 O’Nuggets, a humongous pack contains 9 O’Nuggets, and a colossal pack contains 20 O’Nuggets. What is the largest number of Chicken O’Nuggets that is impossible to order?

### Plane passengers

There are one hundred people waiting to board a plane with one hundred seats. However, the first passenger to board has forgotten their seat number and decides to occupy a random seat. Each subsequent passenger takes their assigned seat if it is available and otherwise takes a random unoccupied seat. What is the probability that the last passenger to board finds their seat unoccupied?

### Crime investigation

While questioning a witness, a judge is only allowed to ask questions which are to be answered ‘yes’ or ‘no’. The judge has carefully calculated that, as long as the witness answers every question truthfully, then she can solve the case in no more than 91 questions. One or more of these questions may depend on the answers to previous questions. Show that the judge can solve the case in no more than 105 questions if it is known that the witness may answer at most one question falsely.



### A facetious function

The following question appeared in the 1988 International Mathematical Olympiad which was held in Canberra.

A function  $f$  defined on the positive integers and taking positive integer values is given by

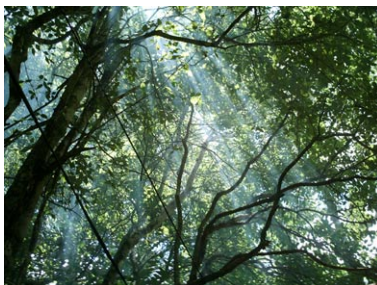
$$\begin{aligned} f(1) &= 1, \\ f(3) &= 3, \\ f(2n) &= f(n), \\ f(4n+1) &= 2f(2n+1) - f(n), \\ f(4n+3) &= 3f(2n+1) - 2f(n), \end{aligned}$$

for all positive integers  $n$ . Determine with proof the number of positive integers less than or equal to 1988 for which  $f(n) = n$ .

### A mathematician is lost in the woods...

- (1) A mathematician is lost in the woods. He knows that its area is  $A$  square kilometres and that it has no holes. Show that he can escape by walking no more than  $2\sqrt{\pi A}$  kilometres.

- (2) A mathematician is lost in the woods. He knows that its area is  $A$  square kilometres and that it is convex in shape. Show that he can escape by walking no more than  $\sqrt{2\pi A}$  kilometres.
- (3) A mathematician is lost in the woods. He knows that its area is  $A$  square kilometres and that it is convex in shape. Show that if he is allowed to consult a person who knows the shortest way out, then he can escape by walking no more than  $\sqrt{A/\pi}$  kilometres.
- (4) A mathematician is lost in the woods. He knows that it is in the shape of a half plane and that he is precisely one kilometre away from the edge of the woods. Show that he can escape by walking no more than 6.4 kilometres.
- (5) A mathematician is lost in the woods. He knows that it is in the shape of a strip one kilometre wide and infinite in length. Show that he can escape by walking no more than 2.3 kilometres.



### Solutions to Puzzle Corner 2

The \$50 book voucher for the best submission to Puzzle Corner 2 is awarded to Natalie Aisbett.

#### Coffee and doughnuts

*Solution by Jeremy Ottenstein:* Let  $C$  and  $D$  be the price in dollars for a coffee and a doughnut, respectively. Let  $B$  and  $G$  be the number of boys and girls, respectively. From the information given in the problem, we have the equation

$$BC + GD = BD + GC + 1 \Rightarrow (B - G)(C - D) = 1.$$

This implies that  $B - G$  is a divisor of 1 so that  $B - G = \pm 1$ . Since there are more girls than boys in the class, it follows that  $G = B + 1$ . Apart from the obvious fact that  $B$  and  $G$  must also be nonnegative integers, this is all that can be determined about  $B$  and  $G$ .

#### Solitaire



*Solution by Christopher Chen:* Let us say that the game can be solved if it is possible to end with one piece remaining on the board. We conjecture that the game can be solved if and only if  $n$  is not a multiple of 3.

Part 1: The game cannot be solved when  $n$  is a multiple of 3.

Start by labelling each square of the board with the sum of its coordinates taken modulo 3. Each square will be given a label 0, 1 or 2 with the property that any three consecutive squares have distinct labels. Let  $X_0$ ,  $X_1$  and  $X_2$  denote the



number of pieces on squares labelled by 0, 1 and 2, respectively. If  $n$  is a multiple of 3, it is easy to see that at the beginning of the game,  $X_0 = X_1 = X_2$ .

Observe that whenever a move is made, we lose two pieces on squares of different labels, but gain a piece on a square of the third label. Therefore, the triple  $(X_0 \bmod 2, X_1 \bmod 2, X_2 \bmod 2)$  is invariant under any move. Note that our initial value is  $(0, 0, 0)$  or  $(1, 1, 1)$  while our desired final value should be  $(1, 0, 0)$ ,  $(0, 1, 0)$  or  $(0, 0, 1)$ . Therefore, it is impossible to end with only one piece remaining on the board when  $n$  is a multiple of 3.

Part 2: The game can be solved when  $n$  is not a multiple of 3.

If  $n \equiv 1 \pmod{3}$ , then it is possible to remove the pieces on the perimeter of the original square, leaving us with an  $(n-2) \times (n-2)$  block. We note that  $n-2 \equiv 2 \pmod{3}$ .

If  $n \equiv 2 \pmod{3}$ , then it is possible to remove the pieces on the perimeter as well as those pieces which are adjacent to those on the perimeter, leaving us with an  $(n-4) \times (n-4)$  block. We note that  $n-4 \equiv 1 \pmod{3}$ .

Therefore, by alternately applying the above two procedures, we will be left with either the  $n=1$  case or the  $n=2$  case, both of which can obviously be solved.

### Glass half full

*Solution by Gerry Myerson:* The key is to use the symmetry of the glass — no extra equipment is required! Simply tilt the glass as far as you can without spilling any water. If the bottom of the glass is not covered entirely by water, then the glass is less than half full. If the bottom is covered with water to spare, then the glass is more than half full. And, of course, if the bottom is covered with no water to spare, then the glass is exactly half full.

### Secret salaries

*Solution by Peter Pleasants:* The solution involves six pieces of paper. Alice writes the same random number on two pieces of paper and passes one to Bob. On a third piece of paper, Bob adds his salary to the number he received from Alice and passes the result to Clare. On a fourth piece of paper, Clare adds her salary to the number she received from Bob and passes the result to Alice. On a fifth piece of paper, Alice adds her salary to the number she received from Clare and passes the result to Bob. On the sixth and final piece of paper, Bob adds his salary to the second number he received from Alice and passes the result to Clare.

Let  $R$  be Alice's random number and  $A$ ,  $B$ , and  $C$  be Alice, Bob, and Clare's salaries, respectively. Then after the whole procedure, each person has two pieces of paper with the following numbers.

- Alice:  $R$  and  $R + B + C$
- Bob:  $R$  and  $R + A + B + C$
- Clare:  $R + B$  and  $R + A + 2B + C$

Now each person need only take the difference between the two numbers that they have. Alice will obtain the sum of Bob and Clare's salaries from which she can easily determine the sum of all three salaries and, hence, the average. Bob and Clare each obtain the sum of all three salaries and may similarly determine

the average. Furthermore, no one can extract any more information about any individual salary.

### Ambulatory ants

*Solution by by Sam Krass:*

- (1) Let  $n$  be the number of ants and  $T$  be the time that an ant takes to complete a lap of the circular path. To an observer who cannot distinguish between the ants, it would appear that when a collision occurs, the ants simply pass by each other, rather than changing directions. Therefore, at time  $T$ , the positions of the ants are identical to their starting positions, perhaps up to some permutation.

However, note that the ants always maintain the same cyclic ordering around the circular path. So at time  $T$ , each ant has been cyclically shifted along  $k$  places from their initial positions for some  $0 \leq k < n$ . Therefore, at time  $nT$ , every ant will be in its starting location.

- (2) *As noted by several astute readers, the problem is not true unless the ants all walk at the speed of one metre per minute. Your humble author wishes to apologise for this omission in the original problem statement.*

- (a) Once again, to an observer who cannot distinguish between the ants, it would appear that when a collision occurs, the ants simply pass by each other, rather than changing directions. Therefore, after two minutes, the positions of the ants are identical to their starting positions, perhaps up to some permutation. However, note that the ants always maintain the same ordering along the stick. Therefore, after two minutes, every ant will be in its starting location.

- (b) Using our earlier observations, we note that if an ant begins at distance  $x$  metres from the left end of the stick, then after one minute, there must be an ant at distance  $1 - x$  metres from the left end of the stick. Since the ants always maintain the same ordering along the stick, every ant is in its starting location after one minute if and only if the initial locations of the ants are symmetric about the centre of the stick.

- (3) Let the ants which begin on the second hand, minute hand and hour hand of the clock be  $S$ ,  $M$  and  $H$ , respectively. We will show that after twelve hours,  $S$  has travelled the greatest number of times around the clock.

The crucial observation is that at no time during the twelve hours does one ant overtake another ant. This is due to the fact that when a faster hand overtakes a slower hand, the ant which was on the faster hand ends up on the slower hand. It is clear that at 1 second after noon,  $S$  has travelled further than  $M$ , who has travelled further than  $H$ . Therefore, at every moment during the twelve hours,  $S$  has travelled at least as far as  $M$ , who has travelled at least as far as  $H$ .

Let the number of revolutions completed by  $S$ ,  $M$  and  $H$  be  $R_S$ ,  $R_M$  and  $R_H$ , respectively. From our earlier observations, it follows that  $R_H \leq R_M \leq R_S$ . Furthermore, since  $S$  cannot overtake  $H$ , we also have the inequality  $R_S \leq R_H + 1$ . However, the three ants together have travelled as far as the three hands of the clock together. Since the second hand performs 720 revolutions, the minute hand performs 12 revolutions and the hour hand performs 1 revolution, we obtain  $R_H + R_M + R_S = 720 + 12 + 1 = 733$ . The

only integral solution for  $(R_H, R_M, R_S)$  which satisfies the above inequalities is  $(R_H, R_M, R_S) = (244, 244, 245)$ . Therefore, the ant which begins on the second hand has travelled the greatest number of times around.

### An unusual identity

*Solution by Natalie Aisbett:* Arrange the numbers in two rows as follows.

$$\begin{array}{cccccc} a_1 & a_2 & a_3 & \cdots & a_n \\ b_1 & b_2 & b_3 & \cdots & b_n \end{array}$$

Suppose that there is a value of  $k$  such that  $n < a_k < b_k$ . Then it is clear that the numbers

$$\begin{array}{cccccc} & & & & a_k & a_{k+1} & \cdots & a_n \\ b_1 & b_2 & \cdots & b_k & & & & \end{array}$$

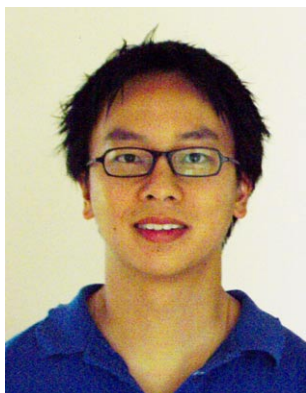
are all at least as large as  $a_k$ . Hence, these  $n + 1$  numbers must belong to the set  $\{n + 1, n + 2, \dots, 2n\}$ , which contradicts the fact that they are distinct. Using analogous arguments, one can show that there is no value of  $k$  such that one of the following occurs:

$$n < a_k < b_k, \quad n < b_k < a_k, \quad n \geq a_k > b_k, \quad \text{or} \quad n \geq b_k > a_k.$$

In other words, we have proven that for every value of  $k$ , the larger of  $a_k$  and  $b_k$  is in the set  $\{n + 1, n + 2, \dots, 2n\}$  while the smaller of  $a_k$  and  $b_k$  is in the set  $\{1, 2, \dots, n\}$ .

Therefore, we have the following chain of equalities:

$$\begin{aligned} & |a_1 - b_1| + |a_2 - b_2| + \cdots + |a_n - b_n| \\ &= [\max(a_1, b_1) - \min(a_1, b_1)] + \cdots + [\max(a_n, b_n) - \min(a_n, b_n)] \\ &= [\max(a_1, b_1) + \cdots + \max(a_n, b_n)] - [\min(a_1, b_1) + \cdots + \min(a_n, b_n)] \\ &= [(n + 1) + (n + 2) + \cdots + 2n] - [1 + 2 + \cdots + n] \\ &= [(n + 1) - 1] + [(n + 2) - 2] + \cdots + [2n - n] \\ &= n + n + \cdots + n \\ &= n^2. \end{aligned}$$



Norman is a PhD student in the Department of Mathematics and Statistics at The University of Melbourne. His research is in geometry and topology, with a particular emphasis on the study of moduli spaces of algebraic curves.



# The style files

## Use the most informative synonym

Tony Roberts\*

Convey the maximum information by using the most precise synonym possible. Avoid unnecessary imprecision.

For example, to write the ‘level’ of some quantity is vague. Better to write the ‘concentration’, or the ‘frequency’, or whatever. The term ‘concentration’ is more specific than ‘level’ and so contains more information. Similarly, in an example from human studies, prefer to write ‘patient’ or ‘gymnast’ instead of the vague ‘subject’. Use synonyms that have more information.

Most terms we use have variants over a wide range of abstraction. Higham [2, Section 4.29] gives these examples:

- graph — function — rational function — polynomial — quadratic — scalar;
- result — theorem — relation — inequality — bound;
- statistic — error — relative error;
- optimum — minimum — global minimum;
- random — normally distributed — normal  $(0, 1)$ .

These lists place the most abstract, general words to the left, and the most concrete, specific words to the right. Prefer the word that is as far to the right as possible as it conveys the most information.

Similarly aim for precision when you choose non-scientific words. Fortunately, the many invaders of England in the past few thousand years left a legacy of a language rich in synonyms: English is one of the most synonym-rich languages. When choosing a synonym, prefer a short, concrete word (often Anglo-Saxon in origin) in preference to a long, abstract word (often of French or Latin origin). Enjoy using uncommon words when connotations associated with the word are just right for you. Then the fewest words will convey the maximum information through their connotations.

Precisely specify forward and backward links. Often authors write ‘the above method’, ‘mentioned above’ or ‘later we see’. Such links internal to the document are vague and imprecise. You, the writer, are referring to something preceding (but not actually above) or following. Such loose references are convenient for writers, but not for readers. You know exactly what and where, but your readers may have to search. Instead be specific.

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Make internal links precise using names or numbering. For examples, the above imprecise links might be more informatively written: ‘the quasi-Newton method’, or ‘mentioned in the Introduction’, or ‘Section 4 shows’.

the ill and unfit choice of words wonderfully obstructs the understanding

Francis Bacon, circa 1600

## References

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Tony Roberts is the world leader in using and further developing a branch of modern dynamical systems theory, in conjunction with new computer algebra algorithms, to derive mathematical models of complex systems. After a couple of decades of writing poorly, both Higham’s sensible book on writing and Roberts’ role as electronic editor for the Australian Mathematical Society impelled him to not only incorporate writing skills into both undergraduate and postgraduate programs, but to encourage colleagues to use simple rules to improve their own writing.



# Communications

## Rolle to Cauchy

J.J. Koliha\* and Peng Zhang\*\*

The mean value theorem is one of the cornerstones of calculus, yet for its proof most calculus books suddenly pull a complicated looking function

$$F(x) = f(x) - f(a) + \frac{f(b) - f(a)}{b - a}(x - a)$$

out of the hat in the quest of reducing the problem to Rolle's theorem on  $[a, b]$ .

Not surprisingly many first year students of calculus feel intimidated by such an *ad hoc* approach, and miss the simple connection between the intuitively more obvious Rolle's theorem, and its more complicated cousins, Lagrange's and Cauchy's mean value theorems.

In this note we offer a delightfully simple do-it-yourself approach to Cauchy's mean value theorem which provides the missing motivation and goes straight from 'Rolle to Cauchy'. We need to assume the following version of Rolle's theorem (see for instance [1, p.255]).

**Theorem 1** (Rolle's theorem). *Let a function  $f: [a, b] \rightarrow \mathbb{R}$  be continuous in  $[a, b]$ , differentiable in  $(a, b)$ , and let  $f(a) = f(b)$ . Then there exists a point  $c \in (a, b)$  such that  $f'(c) = 0$ .*

Now assume that functions  $f, g: [a, b] \rightarrow \mathbb{R}$  are continuous in  $[a, b]$  and differentiable in  $(a, b)$ , and that  $g'(x) \neq 0$  for all  $x \in (a, b)$ . In order to apply Rolle's theorem, we set

$$G(x) = f(x) - Ag(x)$$

with  $A \in \mathbb{R}$  chosen to satisfy  $G(a) = G(b)$ . First we show that  $g(a) \neq g(b)$ . If we had  $g(a) = g(b)$ , then by Rolle's theorem there would exist  $d \in (a, b)$  such that  $g'(d) = 0$ . But this contradicts our assumption about the derivative of  $g$ . Hence  $g(a) \neq g(b)$ . Solving  $G(a) = G(b)$  gives  $A = (f(b) - f(a))/(g(b) - g(a))$ . The function  $G$  satisfies the assumptions of Rolle's theorem, and hence there exists  $c \in (a, b)$  with  $G'(c) = 0$ , that is,

$$G'(c) = f'(c) - Ag'(c) = f'(c) - \frac{f(b) - f(a)}{g(b) - g(a)}g'(c) = 0.$$

Thus we have proved

**Theorem 2** (Cauchy's mean value theorem). *Let functions  $f, g: [a, b] \rightarrow \mathbb{R}$  be continuous in  $[a, b]$ , differentiable in  $(a, b)$ , and let  $g'(x) \neq 0$  on  $(a, b)$ . Then there*

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exists a point  $c \in (a, b)$  such that

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.$$

Setting  $g(x) = x$  in the preceding theorem, we obtain

**Theorem 3** (Lagrange's mean value theorem). *Let  $f: [a, b] \rightarrow \mathbb{R}$  be continuous in  $[a, b]$  and differentiable in  $(a, b)$ . Then there exists a point  $c \in (a, b)$  such that*

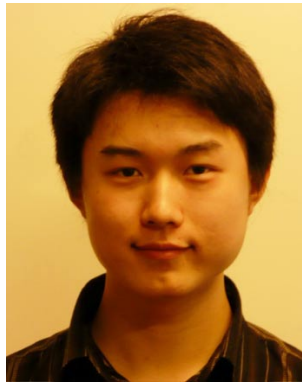
$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

### References

- [1] Thomas, G.B., Weir, M.D., Hass, J.D. and Giordano, F.R. (2005). *Thomas' Calculus*, 11th edn. Pearson/Addison Wesley, Boston.



J.J. Koliha



Peng Zhang

J.J. Koliha is Associate Professor in the Mathematics and Statistics Department of the University of Melbourne. He specialises in Functional Analysis. He says that every year he learns from his students as much as they learn from him.

Peng Zhang is an international student currently in the first year of the Commerce Actuarial Studies at the University of Melbourne. He has been interested in mathematics ever since he could write his own name. He is of Chinese background, born in Beijing.

## **Vacancy: ANZIAM Secretary**

The ANZIAM Secretary, Dr Bill Summerfield, will stand down at the end of the present session (that is, at the conclusion of the 33rd General Meeting) after serving 24 years in the position. This means ANZIAM will require that another member be nominated to be Secretary by that time, to take over for the next session which commences during ANZIAM 2008. The venue for the conference is the Blue Mountain township of Katoomba, NSW, over 3–7 February 2008.

Consequently, ANZIAM seeks expressions of interest from members who are interested in acting as Secretary. Ideally the new occupant will stay in the position for several years. Interested members should, in the first instance, contact Peter Taylor, Chair, ANZIAM (P.Taylor@ms.unimelb.edu.au). Bill is preparing a statement of duties, and will happily provide advice about the requirements of the position (Bill.Summerfield@newcastle.edu.au).

## **AustMS Accreditation**

The secretary has announced the accreditation of:

- Associate Professor Timothy R. Marchant of the University of Wollongong as an Accredited Fellow (FAustMS).



## Cambridge University Press to publish Australian Mathematical Society Journals

Michael G. Cowling\*

The Society has recently signed an agreement with Cambridge University Press (CUP) to publish *The ANZIAM Journal*, the *Bulletin of the Australian Mathematical Society* and the *Journal of the Australian Mathematical Society* for the next five years (2008–2012, inclusive). The Society will continue to manage the editorial side of these Journals — handling submissions and deciding what to publish — while CUP will manage the production (typing, handling proofs, and printing) and distribution. The Society will continue to deal with all aspects of the *Gazette of the Australian Mathematical Society* and the electronic supplement to *The ANZIAM Journal*.

The ground work for this change was carried out by an *ad hoc* committee comprised of Michael Cowling (chair), Peter Hall, Algy Howe, Alf van der Poorten, and Tony Roberts. On the basis of the information provided to it, Council then voted for the move.

We anticipate that this will lead to a gradual increase in circulation of our journals, and hence an improvement in our reputation (and impact factors, and all the other numbers that excite research administrators). With this in mind, we would like to encourage Australian mathematicians to submit more papers to our journals, as on the whole our best submissions come from this country (of course, these will still be subject to the usual refereeing processes). We are also hoping that, as we should be able to profit from the economies of scale that larger publishers than us can obtain, the Society's finances will improve.

As part of the changes, we will move late this year to an electronic manuscript handling system (Tony Roberts has already set up Open Journal System (abbreviated to OJS) for the electronic supplement of *The ANZIAM Journal* and the *Gazette*, and *The ANZIAM Journal* is also experimenting with it), and the Canberra office of the Society will handle any paper submissions and be the first point of contact for CUP in Australia for management matters. The offices in Adelaide, Brisbane and Melbourne that currently produce *The ANZIAM Journal*, the *Bulletin* and the *Journal* will almost disappear (it is possible that one of them will remain to help with the publication of the *Gazette*).

Members who subscribe to one or more of *The ANZIAM Journal*, the *Bulletin*, or the *Journal* will also notice that CUP will distribute the three journals; these will probably arrive from overseas. The major change that authors will notice is the electronic submission. Things will change rather more for the editorial teams (editors and associate editors); the details here are still being worked on, and should be finalised by the Society's Annual Meeting in late September.

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**MISG2007:  
Mathematics and Statistics in Industry Study Group  
Tim Marchant\* and Maureen Edwards\*\***

This year's ANZIAM Mathematics and Statistics in Industry Study Group (MISG2007) was held at the University of Wollongong, 5–9 February 2007. Associate Professor Tim Marchant was the Director, Dr Maureen Edwards the Associate Director while Ms Joell Hall and Ms Sue Denny acted as the Administrators.

The event attracted about 100 delegates, including 20 postgraduate students, who worked on six industry projects. Five of these projects were submitted by Australian companies and one from New Zealand. These projects covered diverse fields such as financial mathematics and statistics, operations research, solid mechanics and mathematical modelling.



From left: Dr Maureen Edwards, MISG Associate Director; Professor Robert McKibben, Invited Speaker; Associate Professor Tim Marchant, Director; and Mr Joe Maisano, Trading Technology Australia

MISG2007 was fortunate in attracting Professor Robert McKibben, from Massey University, as invited speaker. His wealth of experience in Industrial Mathematics resulted in him presenting two very entertaining and stimulating seminars. Thanks also to Dr Mike Camden, from Statistics NZ and Dr Jeff Dewynne, from Oxford University, for their seminars at the student workshop.

MISG2007 was opened by Mr Stephen Lowe, General Manager Trading, Integral Energy and Professor Margaret Sheil, DVC-Research at Wollongong (who is now CEO of the ARC). Thanks to both these individuals for their attendance and comments at the opening ceremony. MISG2007 attracted significant media publicity; an article appeared in the local newspaper, the *Illawarra Mercury*, and the director was interviewed twice on local ABC radio.

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The conference dinner was held at the City Beach function centre, which enjoys spectacular views over Wollongong Harbour and the coastline. Other social events included a cocktail reception on the Monday evening, a student workshop followed by pizza on the Tuesday and of course, the annual volleyball grudge match, Australia vs The World.

Due to the broad range of skills required to tackle modern industrial mathematics problems many high-profile scientists from the Australian and NZ Statistics and Financial Mathematics communities attended MISG2007 as delegates or moderators. If the MISG meeting is to remain relevant and important in the coming years then this multi-disciplinary approach to industrial problem solving needs to continue, with participation at MISG from all the Mathematical Sciences.

Moderation of an MISG project is a challenging task involving problem solving, people management and a very tight deadline. So our thanks and gratitude go to the moderators of each of the MISG2007 projects. Following on the tradition from MISG2006 in NZ, each project also included a postgraduate student as moderator.



Delegates from MISG2007 working on the Tranpower NZ Industry Project

See our website <http://www.misg.math.uow.edu.au> for the industry project descriptions from MISG2007, the equation-free summaries of the projects and the outcomes, and also pdf files of the project reports, as they become available. The website also provides details of MISG2008, to be held 28 January–1 February 2008. We hope to see everybody at UOW for the 2008 event.

## Report on ICIAM 2007 in Zurich

Neville de Mestre\*

There were 35 participants from Australia and New Zealand out of a total of 3200 registrations at ICIAM 2007 in Zurich, Switzerland from 16 to 20 July.

The organisation was first rate with the website and the program book co-ordinated by Ross Moore (Macquarie University), who was on secondment to the Congress Director, Professor Rolf Jeltsch. The book contained a listing of all the 3000 plus lectures, mini-symposia and posters in a form easy to follow. At one particular time during the week there were 71 parallel lectures, so it was no easy task for any participant to choose from the many interesting ones on offer. Even the principal invited speakers were scheduled in groups of three.

The lectures were held in 10 separate buildings on the campus of ETH Zurich, but the signs around the campus and the programme book containing maps made it easy to find the appropriate rooms. The weather was unusually hot for Zurich, and lack of air-conditioning in some lecture rooms made it uncomfortable at times.

Following Noel Barton's initiatives from Sydney 2003, there were a number of industry days and embedded meetings.

Four of the applied mathematics prizes were awarded to Heinz Engel (Austria), Joe Keller (USA), Felix Otto (Germany), Gil Strang (USA), while a fifth was shared by Ingrid Daubechies (USA) and Peter Deuffhard (Germany). The 50th Prandtl lecture was given by Tim Pedley (Cambridge) on the fluid mechanics of swimming micro-organisms in which reference was made to the original work in this field by Sir James Lighthill and John Blake (formerly an Adelaide graduate and Professor at Wollongong). There was also a special lecture on the Swiss mathematician Leonhard Euler as 2007 marked the 300th anniversary of his birth. His contribution to many areas of mathematics was phenomenal, particularly as he was practically blind during the period of his greatest output.

A feature of the Congress was the free transport on the city's trams, buses and trains provided by the organisers for every participant for the whole week. Zurich was a great venue. The next ICIAM Congress is in Vancouver in 2011.

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# Technical papers

## From Heron's formula to a characteristic property of medians in the triangle

Árpád Bényi\* and Ioan Caşu\*\*

### Abstract

In an arbitrary triangle, the medians form a triangle as well. We investigate whether this simple property holds for other intersecting cevians as well, and show that the answer is no.

It is a well-established fact in Euclidean geometry that, given a triangle  $ABC$  with sides  $a$ ,  $b$  and  $c$ , its medians  $m_a$ ,  $m_b$  and  $m_c$  also form a triangle called the median triangle. In fact, it can be shown that the median triangle exists in hyperbolic geometry as well, see [3]. Returning to the traditional setting, the existence of the median triangle can be proved in many ways, in particular, through a Heron-type formula which relates the area of  $ABC$  only in terms of its medians. We have:

$$\text{Area}(ABC) = \frac{4}{3} \sqrt{s(s - m_a)(s - m_b)(s - m_c)},$$

where  $s = (m_a + m_b + m_c)/2$ . Assuming without loss of generality that  $a \geq b \geq c$ , we get  $m_a \leq m_b \leq m_c$ . Furthermore, the existence of the square root gives  $s > m_c$ , which proves that the triangle inequalities are indeed satisfied for the medians. If the expression inside the square root comes out negative, we go beyond the familiar Euclidean plane and end up in a Lorentz or Minkowski plane. A nice discussion of what happens in this case can be found in [2].

The reader might very well ask: is there a bisector triangle or an altitude triangle (in the Euclidean plane) as well? The answer is no, and one can easily construct examples of triangles for which neither the bisectors nor the altitudes satisfy the triangle inequality; see for example [1]. It is as if the medians are 'the chosen ones' to form a new triangle among all intersecting cevians in the given triangle! Could this be true? Our investigations lead us to the surprising answer of yes! We can indeed prove a certain uniqueness property of the median triangle. Moreover, and to add to the surprise, we will use a bit of algebra and analysis in our proof. Because of this, the proof if elementary will seem technical at times. Of course, it would be interesting to know whether any geometric and more intuitive proofs exists as well, but so far we have been unable to find any.

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We start by setting up some notation. Let  $0 < p, q, r < 1$  be three arbitrary numbers. Depending on the context we will use  $|\cdot|$  to denote either the length of a segment or the absolute value of a real number. Given the triangle  $ABC$  with sides  $a, b, c$ , we consider the points  $A' \in (BC)$ ,  $B' \in (CA)$ , and  $C' \in (AB)$  such that  $AA', BB', CC'$  are intersecting cevians, and  $|BA'| = pa$ ,  $|CB'| = qb$ , and  $|AC'| = rc$ . Ceva's theorem then restricts one of the parameters  $p, q, r$  to be dependent on the other two, since

$$\frac{|BA'|}{|A'C|} \cdot \frac{|CB'|}{|B'A|} \cdot \frac{|AC'|}{|C'B|} = 1 \iff \frac{pqr}{(1-p)(1-q)(1-r)} = 1,$$

or

$$r = \frac{(1-p)(1-q)}{pq + (1-p)(1-q)}. \quad (1)$$

We are now ready to state our main result.

**Theorem 1** (Uniqueness of medians). *If for all triangles  $ABC$  in the plane, three given intersecting cevians  $AA', BB', CC'$  form a triangle, then these cevians must be medians.*

Let us first clarify the statement of our theorem. We allow the sides  $a, b$  and  $c$  of the triangle to be arbitrary numbers strictly larger than zero, restricted only by the triangle inequality

$$b + c > a > |b - c|. \quad (2)$$

The parameters  $0 < p, q < 1$ , and in particular  $r$  given by (1), are fixed, that is we require the cevians to cut the three sides of any triangle in the same ratio.

Using the cosine formula in triangles  $ABA'$  and  $ABC$ , we can write

$$|AA'|^2 = c^2 + (pa)^2 - 2c(pa) \cos(\angle B) = c^2 + p^2 a^2 - p(a^2 + c^2 - b^2),$$

or

$$|AA'| = \sqrt{p^2 a^2 - p(a^2 + c^2 - b^2) + c^2}.$$

Similarly, we have

$$|BB'| = \sqrt{q^2 b^2 - q(b^2 + a^2 - c^2) + a^2},$$

$$|CC'| = \sqrt{r^2 c^2 - r(c^2 + b^2 - a^2) + b^2}.$$

We can now reformulate Theorem 1 and concentrate on proving the restated result.

**Theorem 1\***. *Let  $0 < p, q < 1$  be fixed and  $r = r(p, q)$  be defined by (1). If  $|AA'| + |BB'| > |CC'| > ||AA'| - |BB' ||$  for all  $a, b, c > 0$  satisfying (2), then  $p = q = r = 1/2$ .*

*Proof.* We simplify further our statement. Let  $u = b/a, v = c/a$  and define

$$S = \{(u, v) \in (0, \infty) \times (0, \infty) : u + v > 1 > |u - v|\}.$$

For fixed  $0 < p, q < 1$ , and  $(u, v) \in S$ , define

$$\begin{aligned} \phi(p, q, u, v) &= (pq + (1 - p)(1 - q)) \\ &\quad \times (\sqrt{p^2 - p(1 + v^2 - u^2)} + v^2 + \sqrt{q^2 u^2 - q(1 + u^2 - v^2)} + 1), \\ \psi(p, q, u, v) &= \sqrt{(1 - p)^2(1 - q)^2 v^2 - (pq + (1 - p)(1 - q)) \\ &\quad \times (1 - p)(1 - q)(u^2 + v^2 - 1) + u^2(pq + (1 - p)(1 - q))^2}, \\ \delta(p, q, u, v) &= (pq + (1 - p)(1 - q)) \\ &\quad \times |\sqrt{p^2 - p(1 + v^2 - u^2)} + v^2 - \sqrt{q^2 u^2 - q(1 + u^2 - v^2)} + 1|. \end{aligned}$$

Note that if we factor out  $a^2$  in the expressions under the square roots that define  $|AA'|$ ,  $|BB'|$ ,  $|CC'|$ , substitute the expression (1) defining  $r$  in the length formula of  $|CC'|$ , and multiply through with  $pq + (1 - p)(1 - q)$  in the triangle inequality satisfied by the intersecting cevians, we arrive at the following.

*Claim.* If  $\phi(p, q, u, v) > \psi(p, q, u, v) > \delta(p, q, u, v)$  for all  $(u, v) \in S$  then  $p = q = \frac{1}{2}$ .

To prove this claim we use an invariance property of the set  $S$  and some symmetry properties of the functions  $\phi$ ,  $\psi$  and  $\delta$ .

First, note that  $S$  is an open strip in the first quadrant of the  $uv$ -plane that is bordered by (and does not include) the slanted lines  $v = u + 1$ ,  $v = u - 1$ , and  $v = 1 - u$ . The first two of the border lines intersect the third one at the points  $P = (0, 1)$  and  $Q = (1, 0)$ , which belong to the boundary  $\partial S$  of our strip (see Figure 1).

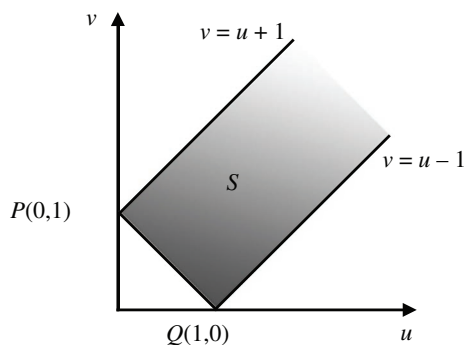


Figure 1.

It is easy to show that  $S$  satisfies the following invariance property

$$(u, v) \in S \leftrightarrow \left(\frac{1}{u}, \frac{v}{u}\right) \in S. \tag{3}$$

Second, a series of elementary, yet tedious, computations give the symmetry properties

$$\begin{aligned}\phi\left(1-q, 1-p, \frac{1}{u}, \frac{v}{u}\right) &= \frac{1}{u}\phi(p, q, u, v), \\ \psi\left(1-q, 1-p, \frac{1}{u}, \frac{v}{u}\right) &= \frac{1}{u}\psi(p, q, u, v), \\ \delta\left(1-q, 1-p, \frac{1}{u}, \frac{v}{u}\right) &= \frac{1}{u}\delta(p, q, u, v).\end{aligned}\tag{4}$$

The upshot of the invariance and symmetry properties (3) and (4) is that for any equation satisfied by the pair  $(p, q)$  we get another ‘dual equation’ by replacing  $(p, q)$  with  $(1-q, 1-p)$ . Clearly,  $\phi$ ,  $\psi$ , and  $\delta$  are continuous functions (of the variables  $u, v$ ) on  $S$ . If we pass to the limit, as  $(u, v) \rightarrow P \in \partial S$ , the second inequality in the condition of the Claim becomes

$$\begin{aligned}(1-p)(1-q) &= \psi(p, q, 0, 1) \geq \delta(p, q, 0, 1) \\ &= (pq + (1-p)(1-q))\left|\sqrt{p^2 - 2p + 1} - 1\right|,\end{aligned}$$

or, after simplifying,

$$(1-p)^2(1-q) \geq p^2q.\tag{5}$$

The dual inequality of (5) is

$$q^2p \geq (1-q)^2(1-p).\tag{6}$$

Similarly, when  $(u, v) \rightarrow Q \in \partial S$ , the first inequality in the condition of the Claim gives

$$(pq + (1-p)(1-q))(p + \sqrt{q^2 - 2q + 1}) \geq pq + (1-p)(1-q),$$

or

$$p \geq q.\tag{7}$$

Note that the dual of (7) gives the same inequality.

We are now very close to the conclusion. Let  $s = p/(1-p)$  and  $t = q/(1-q)$ . From (5), (6), (7) we have

$$s^2t \leq 1, \quad st^2 \geq 1, \quad \text{and} \quad s \geq t,$$

or

$$\frac{1}{st} \geq s \geq t \geq \frac{1}{st}.$$

Thus,  $s = t = 1$  which is equivalent to  $p = q = \frac{1}{2}$  (and hence also  $r = \frac{1}{2}$ ). The proof is complete.

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## How to multiply and divide triangles

Maurice Craig\*

### Introduction

‘Rational’ trigonometry [4] invokes a novel vocabulary. New words promote concepts that, although easily expressible in Euclidean terms, arguably simplify the subject. Thus Pythagoras’ Theorem, expressed in ‘quadrances’ (squared lengths), reduces to a linear equation.

Pedagogical debate need not inhibit other inquiry. Here I relate triangles, whose sides have integral quadrances, to positive binary quadratic forms with integer coefficients. Geometry makes no special study of triangles with commensurate sides, nor do some of the simplest triangles (e.g. the right-angled isosceles) fit that type. But there is an extensive theory of integral quadratic forms and hence, implicitly, of triangles with integer quadrances.

Section 2 recalls basic notions about quadratic forms, while Section 3 sets up their correspondence with triangles. The fourth section then examines the trigonometric counterpart of composition identities.

### Quadratic forms

A real binary quadratic form is an expression  $f(x, y) = ax^2 + bxy + cy^2$ . Completing the square shows that  $f(x, y)$  is positive definite if, and only if, the leading coefficient  $a > 0$  and the discriminant  $b^2 - 4ac = -d < 0$ .

Number theory considers forms whose coefficients  $a, b, c$  and variables  $x, y$  are integers. Suppose  $g(x, y) = f(\alpha x + \beta y, \gamma x + \delta y)$ , where  $\alpha, \beta, \gamma, \delta$  are integers with  $\alpha\delta - \beta\gamma = \pm 1$ . We say  $f$  and  $g$  are equivalent, properly or improperly according as the sign is plus or minus. Properly equivalent forms make up a proper equivalence class (or simply a class) of forms.

The contraction  $(a, b, c)$  is often used. Thus, we can remark that  $(1, 0, 2)$  and  $(2, 0, 1)$  are mutually equivalent, both properly and improperly, although verification might mean restoring the suppressed variables. Every form considered below is positive, and hence [3, Section 92] properly equivalent to a reduced form: one with  $|b| \leq a \leq c$ . This reduced form is unique if, when  $|b| = a$  or  $a = c$ , we require further that  $b \geq 0$ . (Caution: [2], [3] treat forms  $(a, 2b, c)$ .)

Several different (and hence properly inequivalent) reduced forms can have the same  $d$ . For instance, the reduced forms with  $d = 23$  are  $(1, 1, 6)$ ,  $(2, 1, 3)$  and  $(2, -1, 3)$ . Accordingly, three classes comprise forms with discriminant  $(-23)$ .

Classes dominate the theory of Diophantine equations  $f(x, y) = M$ . Equivalent forms take the same values for integer  $x$  and  $y$ , so the equations for a whole class can be treated together. (See also [3, Section 86(ii)].) Moreover, solutions for composite  $M$  are built up from solutions of equations  $g(x, y) = N$ . Here,  $N$  denotes a divisor of  $M$ , while  $g$  represents a form with the same discriminant as  $f$ , though not necessarily in the same class.

This reduction, from  $M$  to  $N$ , depends on composition of forms. For a comprehensive treatment, see [1], [2], and especially [3, Section 111]. The basic rule for performing composition reads

$$(a, b, Ac)(A, b, ac) = (aA, b, c).$$

Thus the compound of two forms, respectively representing  $a$  and  $A$ , will itself represent the product  $aA$ . All three forms in this shorthand equation have the same central coefficient  $b$ , and the same discriminant  $b^2 - 4aAc = -D$ . Rewritten with the variables restored, the result is  $F(x, y)G(s, t) = H(X, Y)$ , where  $X = xs - cyt$ ,  $Y = axt + Ays + byt$  and  $F(x, y) = ax^2 + bxy + Acy^2$  etc.

**Example 1.** The composite of  $(2, 1, 3)$  with  $(2, -1, 3)$  is  $(1, 1, 6)$ . For,  $(2, -1, 3)$  is properly equivalent to  $(3, 1, 2)$ , so the formula produces  $(6, 1, 1)$ , in the same class as  $(1, 1, 6)$ . More generally,  $(a, b, c)(c, b, a)$  is a principal form, representing unity. To ‘divide’ by  $(a, b, c)$  is to multiply by  $(c, b, a)$ , in its ‘reciprocal’ class.

### Forms and triangles

Let  $U, V, W$  and  $u, v, w$  be the angles and respective opposite sides of a triangle. The connection with forms flows from the cosine rule  $w^2 = u^2 + v^2 - 2uv \cos W$ , reformulated in [4] as the ‘cross law’. Thus, after rearrangement, squaring and use of the Pythagorean identity relating sine and cosine, we arrive at

$$(u^2 + v^2 - w^2)^2 - 4u^2v^2 = -(2uv \sin W)^2.$$

Consequently,  $(u^2, u^2 + v^2 - w^2, v^2)$  is a positive definite (real) binary quadratic form. Our main result reformulates this finding more exactly.

**Proposition 1.** *The (row-vector) mapping  $[a, b, c] \mapsto [p, q, r] = [a, c, a - b + c]$  defines a 3-to-1 correspondence between positive definite integral forms  $(a, b, c)$  and triangles whose sides, taken anti-clockwise, have integer quadrances  $p, q, r$ .*

*Proof.* We have  $r = f(1, -1) > 0$ . Moreover (with  $b^2 - 4ac = -d$ ),

$$d = (\sqrt{p} + \sqrt{q} + \sqrt{r})(-\sqrt{p} + \sqrt{q} + \sqrt{r})(\sqrt{p} - \sqrt{q} + \sqrt{r})(\sqrt{p} + \sqrt{q} - \sqrt{r}) > 0.$$

The first factor is positive, as are at least two others, e.g. the last two, if  $p = \max(p, q, r)$ . So all factors are positive, ensuring that every putative side is less than the sum of the other two.

Triangles with sides  $p, q, r$  and the same orientation are necessarily congruent. Choosing anti-clockwise orientation as standard lets us associate, with each form  $(a, b, c)$ , a triangle — having integer quadrances — unique up to congruence.

Conversely, given a triangle  $T$  whose integer quadrances, in anti-clockwise order, are  $p, q, r$ , by taking  $[a, b, c] = [p, p + q - r, q]$  we obtain a positive form  $(a, b, c)$

that maps to  $T$ , whence the mapping is surjective. Three such forms arise, depending on whether  $a = p$ ,  $q$  or  $r$ . (In exceptional cases the forms coalesce and the correspondence is no longer 3-to-1.)

**Example 2.** The first part of the proof follows also from the stock trigonometric formulae  $u = v \cos W + w \cos V$  etc.

**Example 3.** Let the quadrances for  $T$  satisfy  $p \leq q \leq r$ . If no interior angle is obtuse then  $(p, p + q - r, q)$  is a reduced form. For  $p + q - r \geq 0$  (Pythagoras!). However, the triangle with sides 3, 4, 6 gives  $(9, -11, 16)$ , not a reduced form.

### Composition of triangles

A further development now intervenes, annulling the ambiguity in Proposition 1. For, the three forms  $(p, p + q - r, q)$ ,  $(q, -p + q + r, r)$ ,  $(r, r + p - q, p)$  are easily shown properly equivalent. Hence the mapping defined above, while 3-to-1 as a correspondence between forms and triangles, is potentially a 1-to-1 mapping of classes. To realise this potential, we define triangle classes accordingly: call triangles equivalent if their associated forms are so. Corresponding to the equal discriminants of equivalent forms, we have equal areas (or ‘quadreas’, in rational trigonometry) for equivalent triangles.

Composition of forms implies a law for compounding triangles. Thus, from the triangles with quadrances  $\{2, 3, 4\}$  and  $\{2, 3, 6\}$ , Example 1 produces a further triangle, their compound, with the same quadrea but with quadrances  $\{1, 6, 6\}$ . This illustration typifies composition of two triangles with integer quadrances and the same quadrea.

**Example 4.** Let  $R$  be the interior angle opposite  $r$  for  $(a, b, Ac)$ . Define  $R'$ ,  $R''$  similarly for  $(A, b, ac)$  and  $(aA, b, c)$ . By the cosine rule we find  $R = R' = R''$ .

Instances like  $(1, 0, 1)$  and  $(1, 1, 1)$  will alert experts that integer-quadrance triangles must relate somehow to the point-lattice representation of quadratic forms [2, Section 121–124]; [3, Section 120]. The recondite properties of rational trigonometry’s simplest figures show, nevertheless, how thin is the divide between elementary and advanced mathematics. Trigonometry was not meant to be easy!

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# Book reviews

## **Yearning for the impossible: The surprising truths of mathematics**

John Stillwell

A. K. Peters, Ltd., 2006, ISBN 156881254X

In a nutshell: This is a great book, read it!

Many readers of the *Gazette* will be familiar with the beautiful books by author John Stillwell, such as 'Mathematics and its History', 'Geometry of Surfaces' and 'Classical Topology and Combinatorial Group Theory'. Unlike the texts that John Stillwell is famous for among mathematicians, 'Yearning for the Impossible' is aimed at a general audience. However, this does not mean that it is filled with trivialities, or is devoid of real mathematics, as is true for many popularisations of mathematics. On the contrary, realising that there is still 'no royal road to mathematics', the author's aim is to strike just the right balance between mathematical content and conveying the beauty of mathematics. He achieves this by writing so that 'readers with a good mathematical background from high school should be able to appreciate all, and understand most, of the ideas in the book'. In this he succeeds brilliantly, and, furthermore, I cannot imagine anybody with a more sophisticated background in mathematics not also enjoying this book.

In the preface of his book Stillwell writes:

There are many instances of apparent impossibilities that are important to mathematics, ... Mathematical language is littered with pejorative and mystical terms — such as irrational, imaginary, surd, transcendental — that were once used to ridicule supposedly impossible objects. And these are just terms applied to numbers. Geometry also has many concepts that seem impossible to most people, such as the fourth dimension, finite universes, and curved space — yet geometers (and physicists) cannot do without them ... Mathematics is a story of close encounters with the impossible because *all its great discoveries are close to the impossible*. The aim of this book is to tell the story, briefly and with few prerequisites, by presenting some representative encounters across the breadth of mathematics.

The different chapters of the book are entitled: 'The Irrational'; 'The Imaginary'; 'The Horizon' (projective geometry); 'The Infinitesimal'; 'Curved Space'; 'The Fourth Dimension'; 'The Ideal' (numbers beyond numbers); 'Periodic Space' (Escher worlds in mathematics); and 'The Infinite'. Looking at these titles many readers of the *Gazette* will think: 'Been there, done that, know everything there is to know and can be said to a lay person about these things'. While it is true that many of the topics listed here have been written about by many other popularisers of mathematics, John Stillwell manages to find many, many new and interesting angles to keep even experts entertained.

Here are just a few details to whet your appetite. 'The Irrational' is a beautiful exposition of the basics of irrational numbers, which includes the Pythagorean theory

of musical harmonies based on rational numbers, the problems this theory encounters (the Pythagorean comma), and how these problems are naturally overcome by introducing the equal temperament which is based on the irrational frequency ratio of  $\sqrt[12]{2}$ . This chapter also includes some of the gems produced by the theory of continued fractions, a beautiful topic which seems largely forgotten. The chapter ‘Curved Space’ starts out as follows:

People of ancient and medieval times are often said to have believed that the earth was flat, a belief supposedly overthrown by Christopher Columbus. This is a myth. *Not only did the ancients know that the earth was round, they believed that space was round too — an idea that seems impossible to most people today.*

Sound interesting? What do you expect to read about in a chapter entitled ‘The Fourth Dimension’? Quaternions?

The book itself, a hardcover edition, is very well produced and is a pleasure to browse around in. Most pairs of facing pages contain at least one graphical element; unlike most mathematical textbooks, the original illustrations were done by someone who knows what they are doing; and the layout is open and uncluttered.

To summarise: This is a great book, read it!

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### **Applications of abstract algebra with Maple and Matlab 2nd edition**

Richard E. Klima, Neil P. Sigmon and Ernest L. Stitzinger

CRC Press, 2006, ISBN 978-1-5848-8610-5

There is much to admire and like about this book. Its robust and expert use of Maple and Matlab — the leading software for symbolic and numerical work respectively, the breadth of material covered, and the generous exercises all mark it out as deserving high praise.

That being said, I have a few caveats with this book. First, it is not a standard mathematics text, where a theory is carefully built up with axioms, lemmas, propositions, theorems and corollaries, all with proofs, but rather a book which shows how Maple and Matlab can be used to elucidate and illustrate algebraic material. Second, possibly because of the first point, a somewhat cavalier attitude has been taken to some of the topics: there are places in the book where more explanation and a greater attention to detail would have been appropriate. For this reason the book is unlikely to be used as a text in a standard course, but rather as an adjunct to a course in modern applications of algebra.

I will enlarge on these points below.

### Material covered

The authors cover a brave range of material: block designs and difference sets; error-correcting codes — including Hamming codes, Reed–Muller Codes, BCH codes and Reed–Solomon codes; classical cryptography — shift, affine, Vigenère and Hill ciphers; RSA and ElGamal cryptosystems; elliptic curves and elliptic curve cryptosystems; the Advanced Encryption Standard; Burnside’s theorem and Pólya’s counting theorem; graph theory and graph counting.

Chapter 1 discusses ‘Preliminary Mathematics’; mainly groups, rings and finite fields and the implementations of finite fields in Maple and Matlab. The material here is standard, and treated briskly, but with due care. Curiously, there is no mention of the fundamental fact that any two fields of order  $p^n$  are isomorphic. The treatment of fields in Maple relies on constructing powers (modulo the defining polynomial) of a primitive element. This certainly is how one might generate a small field by hand, but surely it would be both more standard and lead to greater power if the authors used Maple’s own finite field package `GF`.

Chapter 2 investigates block designs, Hadamard matrices and difference sets. Some fundamental results are given (but not the Bruck–Ryser–Chowla theorem). I think the authors missed an opportunity here to include some material on finite projective planes.

Chapters 3, 4 and 5 present some elementary material from the theory of error-correcting codes. Specifically: linear codes, Hamming and BCH codes, Hadamard codes, Reed–Muller and Reed–Solomon codes. Although perfect codes are mentioned, the Golay codes are not. The authors provide only the decoding of Reed–Muller codes using Hadamard matrices; majority logic decoding is not mentioned. In the sections on basic linear codes, Hamming codes and Reed–Muller codes, Matlab is used appropriately; its matrix handling used efficiently and well. For the BCH and Reed–Solomon codes, whose definition requires some symbolic algebra, the best the authors can do with Matlab is to call Maple commands from within it. I wonder if it might simply have been better to admit that here Matlab is the wrong tool for the job. In fact of course Reed–Solomon codes can be implemented very efficiently in Matlab, but not in the way that the authors have described them. This is one of the problems faced by using both Maple and Matlab — the intersection of problems efficiently solvable by both is quite small.

In Chapter 6 we move into ‘Algebraic Cryptography’: shift and affine ciphers, and the Hill matrix cryptosystem. The mathematics here is simple, but entertaining, and both Maple and Matlab are appropriately used. Chapter 7 is entirely devoted to the Vigenère cipher and its cryptanalysis with the index of coincidence. I think that an entire chapter devoted to the Vigenère cipher is too much. This cipher is cryptographically trivial, and although some discussion of its cryptanalysis is necessary, I would rather more time be spent on modern cryptosystems. In particular, the RSA and ElGamal cryptosystems are presented purely as algebraic constructs.

Chapter 7 discusses the RSA cryptosystem and the Diffie–Hellman key exchange — both vital topics in modern cryptography. Here Maple is used entertainingly to illustrate RSA, using commands which transfer a string of upper case characters to and from a large integer, by treating each letter’s place in the alphabet as a ‘digit’ in base 100. As with many chapters in the text, this is a place where Matlab is

unsuited for the task, and most of the Matlab commands in this chapter are calls to the Maple kernel. This chapter also contains ‘Notes’ on integer factorisation, primality testing, digital signatures and modular exponentiation. With the exception of the last, all these topics could have benefited from greater detail. Space would have been better used in this book by eliminating Matlab and enlarging these important topics.

Elliptic curves and associated cryptosystems are the topic of Chapter 8 — a change from the first edition of this book, where elliptic curves were only part of a chapter. A starting point is the ElGamal cryptosystem, but the authors miss out on the opportunity to introduce primitive roots. They do not in fact spend any time discussing the base  $a$  in

$$a^n \pmod{p}$$

where  $n < p$  is randomly chosen. To their credit, the system is first introduced over general Abelian groups, but even so too much detail is omitted. Elliptic curves are first introduced in the Cartesian plane and the standard geometric construction is given for point addition and doubling. Then the construction is given algebraically in terms of the point coordinates. This is all very well, but surely a few lines explaining how the point addition formulas are derived from the geometric construction could have been included. Also, a brief nod in the direction of elliptic functions as a basis for the entire theory would have been appropriate. If Pollard’s  $p - 1$  method had been introduced in the previous chapter, this would have been an excellent place for Lenstra’s elliptic curve factorisation method.

Chapter 10 is devoted to the Advanced Encryption Standard; the Rijndael cryptosystem. The trouble here is that because of the authors’ clumsy implementation of finite fields, much of the advantages of using Maple to illustrate the AES are lost. It may have been better to describe the AES, but use Maple (or Matlab) to illustrate a simpler version, possibly that of Musa *et al.* [1].

One major oversight in all of Chapters 8–10 is that no discussion of weaknesses of any of these cryptosystems is given. There is no discussion about their security, attacks against these systems, and how such attacks can be subverted. This is disingenuous, as it gives the impression that the security is based only on the difficulty of factoring or of discrete logarithms, and not on how they are used.

Chapter 11 considers enumeration; specifically Burnside’s theorem and Pólya’s enumeration theorem. This chapter is really well done; with excellent examples and a careful and considered approach to the subject. In the book’s introduction, the authors state ‘... we should also note that Chapter 11 has always been one of our and our students’ favorites’. I would agree in the sense that this is one of the best chapters in the book, from the point of view of the excellence of its mathematics, and of the use of software. Chapter 12 continues this subject and provides an example of the use of enumeration techniques to graph counting.

### Exercises

An otherwise excellent textbook can fall on the basis of poorly written exercises. I am pleased to report that this is not the case here — the authors are clearly meticulous in their teaching, and the exercises all through the book are models of their kind. Each chapter finishes with three groups of exercises, headed ‘Exercises’,

‘Computer Exercises’ and ‘Research Exercises’. The last are invitations for the student to explore the history and uses of the chapter’s material.

### Software used

The authors have chosen Maple and Matlab. In the first edition of this book, Maple alone was used. I expect that the authors decided to include Matlab because it is the most popular numerical software currently available. However, much of the material in this book is concerned with algebraic manipulation, or arithmetic on arbitrarily large integers — both are topics not handled by Matlab! This means that much of the Matlab code samples are simply commands which call Maple code, using a Maple kernel embedded within Matlab. For example, the Maple command given for computing a discrete logarithm is given as

```
> mlog(1438, 256, 8383);
```

and the corresponding Matlab code is

```
>> maple('mlog', sym('1438'), sym('256'), sym('8383'))
```

I think the authors would have been better to use Mathematica instead of Matlab, or indeed any other system which allows for algebraic manipulation and large integer arithmetic. And I would expect that most academics or students who would acquire a book such as this would have access to at least one of Maple or Mathematica.

I find it odd that there is a great amount of finite fields in the book, but no mention of Maple’s GF package for computations on finite fields. Admittedly, it’s a clumsy implementation, but as it’s standard Maple I think the authors should either have used it, or explained why they decided not to.

Some of the code is unnecessarily clumsy and obtuse. For example, the authors take seven lines of Maple code to provide a method which will convert letters of the alphabet to the integer values 0 to 25:

```
> letters := array(0..25, ["A", "B", "C", "D", "E", "F",
> "G", "H", "I", "J", "K", "L", "M", "N", "O", "P", "Q",
> "R", "S", "T", "U", "V", "W", "X", "Y", "Z"]);
> ltable := table():
> for i from 0 to 25 do
>   ltable[ letters[i] ] := i;
> od
```

Since these particular lines of code occur in three separate places in the book, I assume that the authors believe that this is the best way of establishing a correspondence between letters and numbers. These tables and arrays are then used as follows:

```
> message := "ATTACK AT DAWN";
> message := Select(IsAlpha,message);
> ptext := convert(message,list);
> ptext := map(i -> ltable[i], ptext);
```



However, all of this can be done simply in two lines with Maple's `convert,bytes` mechanism, and a shift of 65:

```
> ptext := "ATTACKATDAWN";  
> map(x->x-65,convert(ptext,bytes));
```

Why didn't the authors choose this vastly simpler method? Shorter code is usually best, if it is not unnecessarily obfuscated. It seems as though the authors have not really updated their code since the first edition of the book, to take advantage of the strengths and increased functionality of newer versions of Maple.

### Final remarks

I would probably not use this book as a basis for a course myself, but I do like it as a reference, and as an example of the use of modern software to illustrate mathematics. In terms of the material covered and use of software, I prefer the approach of Trappe and Washington [2].

The use of Matlab, as I have said earlier, is not really suitable for an algebra text. Matlab's strengths are in numerics, not algebra.

However, this text shows a lovely interplay between mathematics and software, and it is the sort of text of which I hope to see much more.

### References

- [1] Musa, M., Schaefer, E. and Wedig S. (2003) A simplified AES algorithm and its linear and differential cryptanalyses. *Cryptologia* **27**, 148–177.
- [2] Trappe, W. and Washington, L. C. (2005). *Introduction to Cryptography with Coding Theory*, 2nd edn. Prentice-Hall.

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# AMSI News

**Philip Broadbridge\***

## Reports and reviews

Some recent Government reports and reviews should be of considerable interest to mathematicians.

World-wide, significant mathematical developments underpin recent advances in applied sciences such as medical imaging, genomic analysis and genetic diagnosis, computer security, telecommunications, financial portfolio design and engineering materials. Hence, it is expected that the demand for expertise in mathematics and statistics will grow. Indeed the recently published DEST SET Skills Audit shows that the period 1997–2005 has already experienced 52% employment growth in the mathematical sciences, compared to 37% over all natural sciences [1].

At the same time, mathematics enrolments in Australia are much lower than a decade ago. The very recently published preliminary report commissioned by the Australian Council of Deans of Science [2] reports a 34% decrease in the EFTSU mathematics teaching load for science students from 1989 to 2005.

The data for the year 2005 show that the decrease over the previous decade in school advanced and intermediate mathematics enrolments has still not bottomed out [3].

I hope that the boost to mathematics funding in the DEST Relative Funding Model, announced in the Federal Budget and discussed in my July column, will eventually help to increase enrolment numbers. As I said then, this increase reflects an acknowledgement by the Government that more money should actually be spent to educate a student in mathematics and statistics. We should be very interested to see if and how this money actually flows on to its intended purpose.

The Council of Australian Governments is currently conducting a National Numeracy Review. This might largely determine guidelines in mathematics education for trainee teachers. AMSI has made an invited formal submission, assisted by Hyam Rubinstein, chair of the 2006 mathematical sciences discipline review. This submission is important, given that the numeracy review committee and its advisory panel contain no academic mathematicians. Independently, the Senate Committee on Academic Standards has recently conducted a hearing. AMSI and ICE-EM have been prominent in their invited verbal presentations to both the numeracy review and the committee on standards.

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### The role of applications in teaching and learning mathematics

These reviews have invoked the question of definition of functional numeracy, and have prompted discussion about the role of applications in teaching and learning mathematics. Some education experts have gone so far as to say that all school mathematics should be taught in the context of applications. However, at university undergraduate level, a study by Sandra Britton [4], found that some students succeed better in applying mathematics after they have learnt it in a decontextualised setting. It is my experience too that students find it harder learning new mathematics as they learn a new application along with it.

Therefore, students who have learnt some new mathematics in a decontextualised setting might be more confident with the techniques so that they are more likely to later use them successfully in applications. I have heard from many experienced facilitators of multi-disciplinary problem-based learning that without guidance, people will not commonly apply mathematical techniques that they have known for less than two years. However, it is also my experience that some students who learn new mathematics in the context of realistic applications are better motivated and have a better long-term appreciation of the material.

Practical utility and intrinsic elegance both play an important motivational role. I feel that a well rounded degree program should contain mathematical modelling courses as well as traditional courses in mathematical methods and deductive reasoning. However, the two distinguishable types of course need not be designed to appear to be so far apart. In my opinion, as a general rule, even in a mathematical modelling course, there should be opportunities to practise mathematical techniques apart from any practical context. On the other hand, assimilation of new abstract concepts is facilitated by referring to realistic applications or to other areas of mathematics that allow a more concrete visualisation.

By making such statements purely in the abstract form, I have immediately broken my own guidelines! However, many other contributors to past issues of the *Gazette* have provided examples of applications-inspired mathematics that are intrinsically interesting from the conceptual point of view.

### Events

From 26 November to 14 December, AMSI and MASCOS will run a joint theme program, 'Concepts of Entropy and their Applications'.

Confirmed speakers so far include I. Müller (Berlin), R. Rubinstein (Technion, Haifa), R. Kleeman (New York), Roderick Dewar (Bordeaux), M. Baake (Bielefeld), A. Guttman (Melbourne), C.A. Hurst (Adelaide), G. Morriss (UNSW), A. Dooley (UNSW), I. Enting (Melbourne) and G. Paltridge (Tas).

The topics, with some flexibility, are: 26 to 28 November, Thermodynamics; 28 to 30 November, Statistical Mechanics; 3 December, Environmental Data Modelling; 4 to 5 December, Dynamical Systems; 6 December, Information Theory; 10 December, Operations Research; 11 to 12 December, Signal Processing; 13 to 14 December, Partial Differential Equations.

On 7 December, AMSI will be hosting a one-day national symposium 'Mathematics Education for 21st C Engineering Students'.

Please consult the AMSI and ICE-EM websites for these and other upcoming events (<http://www.amsi.org.au>, <http://www.ice-em.org.au>).

### References

- [1] Audit of science, engineering & technology skills. (2006). DEST. <http://www.dest.gov.au/NR/rdonlyres/AFD7055F-ED87-47CB-82DD-3BED108CBB7C/13065/SETSAsummaryreport.pdf> (accessed 23 August 2007).
- [2] Dobson, I. (2007). Sustaining Science: University Science in the Twenty-First Century. Australian Council of Deans of Science. <http://www.acds.edu.au/> (accessed 23 August 2007).
- [3] Barrington, F. and Brown, P. (2005). Comparison of Year 12 Pre-tertiary Mathematics Subjects in Australia 2004–2005. Website: <http://www.ice-em.org.au/pdfs/year12maths.pdf> (accessed 23 August 2007).
- [4] Britton, S. (2002). Are students able to transfer mathematical knowledge? In: Boezi, M. (ed.) *Second International Conference on the Teaching of Mathematics*, Crete, July 2002. John Wiley & Sons, New York.



Director of AMSI since 2005, Phil Broadbridge was previously a professor of applied mathematics for 14 years, including a total of eight years as department chair at University of Wollongong and at University of Delaware.

His PhD is in mathematical physics (University of Adelaide). He has an unusually broad range of research interests, including mathematical physics, applied nonlinear partial differential equations, hydrology, heat and mass transport, and population genetics. He has published two books and more than 80 refereed papers, including one with 147 ISI citations. He is a member of the editorial boards of three journals and one book series.



## Completed PhDs

### Queensland University of Technology:

- Recent PhD completions include Paula Denman, Therese Wilson and Jonathan Johansen.

### University of Western Australia:

- Simon Collings, *Frontier points: theory and methods for computer vision*, supervisor: Professor Lyle Noakes (jointly supervised with Computer Science)
- Tomasz Popiel, *Geometrically-defined curves in Riemannian manifolds*, supervisor: Professor Lyle Noakes
- Munzir Said, *Computational optimal control modelling and smoothing for biomechanical systems*, supervisor: A/Professor Leslie Jennings

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## Appointments, departures and promotions

### Flinders University:

- Dr Ray Booth, Dr Theresa Dodds, Professor Peter Dodds, Professor Gopal Gopalsamy and Dr Ananth Rao, mathematicians, have all accepted a voluntary separation from the University as from 31 July 2007. That represents a grand total of approximately 166 years of continuing academic (teaching and research) employment at Flinders. There are three mathematicians remaining as full-time staff at Flinders and it is anticipated that one or two further positions may be created in the near future.

### La Trobe University:

- Lennaert van Veen has left to take up a position in Concordia University, Montreal.

### Swinburne University of Technology:

- Dr Tristan Barnett has been appointed Adjunct Research Fellow. Dr Barnett's research focuses on modelling and statistical analysis in tennis.

### University of South Australia:

- Dr Bronwyn Hajek has been appointed as a Research Associate in Mathematical Modelling.
- Dr Ming-Lu Wu has been appointed as a Lecturer in Financial Mathematics.

### University of Southern Queensland:

- Dr Richard Watson has taken over from Associate Professor Ron Addie as Head of the Department of Mathematics and Computing.

### University of Sydney:

- Dr Nigel O'Brian retired on 23 July.
- Dr William Bertram has been appointed as lecturer.

## Awards and other achievements

### LaTrobe University

- Grant Cairns, of LaTrobe University, has won a Carrick Citation for Outstanding Contributions to Student Learning.

### Honourable Mention for USQ students in Mathematical Contest in Modelling

The USQ undergraduate mathematics and science student team including Adam Blazak, Bernard Stilgoe and Denis O'Sullivan, under the supervision of Dr Sergey Suslov, continued a long-term USQ tradition and participated in the 2007 Mathematical Contest in Modelling. This competition is organized annually by the Consortium for Mathematics and its Applications, an award-winning non-profit organisation whose mission is to improve mathematics education for students of all ages.



From left: Denis O'Sullivan, Bernard Stilgoe and Adam Blazak were supervised by Sergey Suslov

949 teams from 12 countries participated in the contest this year. The USQ team, the only team from Australia, was awarded an Honourable Mention. Overall competition statistics were: 14 Outstanding Winners (1%); 122 Meritorious Winners (13%); 255 Honorable Mentions (27%); 558 Successful Participants (59%).

Congratulation to the USQ team for achieving a ranking in the top 30% of all participants!

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## Conferences and Courses

### Call for abstracts: Bioinformatics Australia 2007

Participants are invited to submit abstracts for oral and poster presentations for the Bioinformatics Australia 2007 conference. Oral presentations are sought for the following five symposium areas: Computational Biology, Methods and Tools in Bioinformatics, Proteomics and Structural Bioinformatics, Biological Systems and Comparative Genomics and Evolution. Two speakers for each symposium will be selected from submitted abstracts. Poster presentations are sought for all bioinformatics-related topics. Early career researchers and students are strongly encouraged to participate.

All participants need to be registered by 24 September 2007 to be included in the conference proceedings.

Please see the conference website for details about submission of abstracts.

Closing date for receipt of abstracts:

Oral presentations: 14th September 2007

Poster presentations: 24th September 2007

Web: <https://www.ausbiotech.org/bioinformaticsaustralia2007>

### Logic Summer School

An Open Invitation to attend the Logic Summer School, The Australian National University, 3 to 14 December 2007.

We call this, 'two weeks of wall-to-wall logic'. The Logic and Computation group at ANU believe that logic is not just about computers, or computer scientists. We say that anything that makes sense can be subjected to logical analysis, which means that this School is attractive to any IT professionals, educators in logic and undergraduate students planning to do research in logic-related fields.

Stijn de Saeger (Japan Advanced Institute of Science and Technology, Summer Scholar in 2007) says:

The Summer School did a very fine job of painting a coherent picture of logic as a field, and how it all hangs together. I never read a text book yet that did that for me, and this is one of the things I valued most about the Summer School.

Courses are taught by some of the world's leading logicians and comprise a blend of practical and theoretical short courses on aspects of pure and applied logic.

This is a unique opportunity to learn more about the field of logic from some of the world's best logicians with state-of-the-art computational science facilities at the ANU.

Presenters: Dr Anbulagan, NICTA; Professor Franz Baader, Technische Universität, Dresden, Germany; Dr Peter Baumgartner, NICTA; Dr Brian Davey, La Trobe University; Dr Rajeev Goré, ANU; Dr Jinbo Huang, NICTA; Dr Tomasz Kowalski, ANU; Dr Jia Meng, NICTA; Emeritus Professor Robert Meyer, Visiting Fellow, ANU; Associate Professor Sophie Pinchinat, Université de Rennes 1, France; Professor John Slaney, ANU/NICTA; Dr Alwen Tiu, ANU.

Is it for you? <http://lss.rsise.anu.edu.au/about>

Program <http://lss.rsise.anu.edu.au/program>

Fees and registration <http://lss.rsise.anu.edu.au/registration>

More information <http://lss.rsise.anu.edu.au>

To discuss this invitation in more detail, including advice on suitable candidacy, please contact Professor John Slaney by email: [John.Slaney@anu.edu.au](mailto:John.Slaney@anu.edu.au)

Read about last year's event: [http://cecs.anu.edu.au/events\\_more?SID=15](http://cecs.anu.edu.au/events_more?SID=15)

The Logic Summer School is supported by The Logic and Computation Program, NICTA and The Research School of Information Sciences and Engineering, ANU.

General enquiries: Email: [lss@mail.rsise.anu.edu.au](mailto:lss@mail.rsise.anu.edu.au); Tel: +61 2 6125 8630; Fax: +61 2 6125 8651.

### **Special Session on University Maths Education, 12–15 December 2007**

We are delighted to announce inclusion of a special session on University Mathematics Education at the 1st Joint Meeting of the American Mathematical Society and NZMS in Wellington in December 2007. Do consider participating. New Zealand is very beautiful at that time of the year!

We're keen to welcome participants with a wide range of specialisations, and build a session of interest to everyone.

There is much to reflect on in the current teaching and learning climate. Sessions are expected to comprise groups of talks that address themes like the following:

- attracting students to the study of mathematics
- maths courses for other professions
- strategies for managing student diversity, at entry level, in particular
  - evolving curricula and assessment
  - effective use of technology for learning mathematics
  - teaching experiments in a range of settings

Your talk should be no longer than 20 minutes duration. Submit an abstract online at <http://atlas-conferences.com/cgi-bin/abstract/submit/catm-01>

If you have any difficulties, contact Tim Passmore at [passmore@usq.edu.au](mailto:passmore@usq.edu.au) Also let Tim know If you need an official letter of invitation for visa purposes.

Register at <http://www.mcs.vuw.ac.nz/~mathmeet/amsnzms2007/>. As the session is organised, details will appear on this conference website, including information on plenaries.

And we look forward to welcoming you!



Special Session Organising Team: Derek Holton (Otago); Bill McCallum (Arizona); Pat Cretchley (University of Southern Queensland); and Tim Passmore (University of Southern Queensland).

### **AAECC-17**

This is a call for papers for 17th Symposium on Applied Algebra, Algebraic Algorithms, and Error Correcting Codes, to be held at the campus of Indian Institute of Science, Bangalore, India, on December 16–20, 2007.

General Chairs: P. Vijay Kumar, Tom Høholdt and Heeralal Janwa.

Technical Program Committee Chairs: Serdar Boztas and Hsiao-feng Lu.

Previously unpublished contributions across a range of topics in applied algebra, error correcting codes, and cryptography are solicited, including: algebra, algebraic computation, codes and algebra, codes and combinatorics, modulation and codes, sequences, and cryptography.

The proceeding will be published in Springer-Verlag's Lecture Notes in Computer Science and will be available at the conference.

Invited speakers: T. Helleseth, R. Koetter, T. Lange, G. McGuire, P. Shankar, and H. Stichtenoth. Also planned is a session on list decoding, organised by Madhu Sudan.

Web: <http://comm.ccu.edu.tw/aaecc17>

### **ANZIAM 2008**

The venue for the ANZIAM conference for 2008 is the Carrington Hotel in Katoomba. Katoomba is situated in the Blue Mountains in New South Wales, about two hours from Sydney by train or car. The conference will run from Sunday 3 February to Thursday 7 February, with the traditional Sunday evening barbecue to start things off.

We are particularly pleased that a very distinguished group of invited speakers have agreed to give talks on their research. Their interests cover a broad spectrum across current areas of applied mathematical research, including dynamical systems, mathematical biology and biomedical modelling, climate research and fluid mechanics. There is also crossover into pure mathematics and theoretical physics through Professor Terence Tao and Professor Sir Michael Berry.

Speakers: Dr Sanjeeva Balasuriya (Connecticut College; J.H. Michell medallist, 2006); Professor Sir Michael Berry (University of Bristol); Professor Matthew England (University of New South Wales); Professor Ben Leimkuhler (University of Edinburgh); Professor Linda Petzold (University of California, San Diego; AMSI Lecturer); Professor Andrew Pullan (University of Auckland); Professor Mick Roberts (Massey University); Dr Yvonne Stokes (University of Adelaide; J.H. Michell medallist, 2007); Professor Terence Tao (University of California, Los Angeles);

Web: <http://www.maths.usyd.edu.au/ANZIAM2008>

or contact Charlie Macaskill ([c.macaskill@maths.usyd.edu.au](mailto:c.macaskill@maths.usyd.edu.au)) or Dave Galloway ([d.galloway@maths.usyd.edu.au](mailto:d.galloway@maths.usyd.edu.au)).

### **New Zealand Institute of Mathematics and its Applications Thematic Programme in Algorithmics, 2008**

In 2008 the six-month program *Algorithms: New Directions and Applications*, funded by the New Zealand Institute of Mathematics and its Applications, will run (for details see <http://www.cs.otago.ac.nz/algorithmics/home/>). The main themes are probabilistic analysis of algorithms, randomized algorithms, approximation algorithms and fixed-parameter complexity, with emphasis on increasing interaction between local and overseas researchers in these areas.

The first event is a meeting to be held at The Crown Hotel in Napier, 18–22 February 2008. We request anyone interested in attending to pre-register as soon as possible at <http://www.cs.otago.ac.nz/algorithmics/activities/fmintreg.html>

Participant numbers are limited to 60. Accommodation in Napier (a seaside city with a population of about 50 000) at that time (summer) is likely to be in short supply. We recommend that accommodation be booked as soon as possible. Possible close-by options are listed on the meeting's website.

The programme will consist of two 50-minute expository talks by each of the invited overseas speakers (including Professor Michael Mitzenmacher (Harvard U.), Professor Dominic Welsh (Oxford U.), Professor Michael Langston (U. Tennessee), and Professor Steve Linton (U. St Andrews)); short (15–20 minute) contributed talks by participants; several group problem sessions. Each problem session will focus on a particular area of algorithmic interest and is aimed at facilitating new collaborative research in New Zealand and between local and overseas researchers.

Please sign up immediately on the above webpage if you are interested.

### **9th World Meeting of the ISBA**

You are warmly invited to the 9th World Meeting of the International Society for Bayesian Analysis (ISBA) to be held at Hamilton Island, Australia, 21–25 July 2008. In the spirit of previous meetings, this conference will offer delegates a programme which embraces many facets of Bayesian Analysis. There will be five keynote speakers, 90 presentations run through three parallel strands and poster evenings. The programme will be combined with lunchtime activities and social events that will allow delegates to explore this truly magnificent location in the heart of Australia's Great Barrier Reef. We look forward to your company.

Web: <http://www.isba2008.sci.qut.edu.au>

E-mail: [isba08@qut.edu.au](mailto:isba08@qut.edu.au)

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### **Visiting mathematicians**

Visitors are listed in the order of the last date of their visit and details of each visitor are presented in the following format: name of visitor; home institution; dates of visit; principal field of interest; principal host institution; contact for enquiries.

- Prof Michael Vaughan-Lee; Christchurch, Oxford; 11 August to 1 September 2007; computational group theory; USN; J.J. Cannon
- Prof Ejaz Ahmed; Statistics, University of Windsor, Canada; approximately 12 August 2007, for three weeks; –; MQU
- Dr Tobias Beck; Austrian Academies of Sciences; 4 August to 2 September 2007; computational algebraic geometry; USN; J.J. Cannon
- Dr Ciprian Coman; University of Glasgow; 25 July to 2 September 2007; –; UWA; Prof Andrew Bassom
- Prof Philip Maini; Center for Mathematical Biology, University of Oxford; 9 Aug to 3 September 2007; –; QUT; –
- Dr Alexander Chervov; Institute for Theoretical and Experimental Physics, Moscow; 23 July to 7 September 2007; quantum groups and their representations; USN; A.I. Molev
- Dr Lassina Dembele; University of Calgary; 16 August to 16 September 2007; modular forms; USN; J.J. Cannon
- Dr Akira Yasuhara; Tokyo Gakugei University; 20 July to 17 September 2007; classical links; USN; J.A. Hillman
- Prof Graeme Segal; All Souls College, Oxford; 29 August to 29 September 2007; cohomology of configuration spaces with coefficients; USN; G.I. Lehrer
- Dr James Parkinson; Cornell University; 4 June to 19 October 2007; buildings and Hecke algebras; USN; D.I. Cartwright
- Mr Mikael Johansson; Mathematisches Institut, Fakultät für Mathematik und Informatik, Jena, Germany; 10 September to 20 October 2007; computational aspects of group cohomology; USN; J.J. Cannon
- Dr Eric Badel; INRA (French National Institute for Agricultural Research): Wood Material Laboratory (LERMAB) – Nancy, France; February to November 2007; –; QUT
- Prof Jiang-Min Pan; University of Yunnan, China; 15 August to 1 November 2007; –; UWA; A/Prof Caiheng Li
- Prof Vyacheslav Futorny; University of San Paolo; 24 September to 3 November 2007; quantum groups and their representations; USN; R. Zhang
- Prof Robert Liebler; Colorado State University, USA; 1 October to 11 November 2007; –; UWA; Prof Cheryl Praeger
- Mr Markus Kirschmer; Lehrstuhl D Für Mathematik; 20 September to 3 November 2007; computational group theory; USN; J.J. Cannon
- Prof John Phillips; University of Victoria, Canada; 1 October to 30 November 2007; analysis and geometry; ANU; Alan Carey
- Prof Elvezio Ronchetti; University of Geneva; 01 November to 30 November 2007; statistical science; ANU; Alan Welsh
- A/Prof Dimitri Leemans; Université Libre de Bruxelles; 15 September to 12 December 2007; computational finite geometry; USN; J.J. Cannon
- Prof Pierre Milman; Toronto; 1 to 15 December 2007; algebraic Aspects of Singularities; USN; L. Paunescu
- Dr Andrea Carbonaro; University of Milano; 15 September to 15 December 2007; analysis and geometry; ANU; Alan McIntosh
- Prof Allen Rodrigo; University of Auckland; 8 August to 15 December 2007; Centre for Bioinformation Science; ANU; Sue Wilson
- Prof Guji Tian; Wuhan Institute of Physics and Mathematics; 17 September to 17 December 2007; applied and nonlinear analysis; ANU; Xu-Jia Wang

- Dr Alex Kitaev; Steklov Mathematical Institute; 25 August 2007 to 31 December 2007; singularities and othe properties of integrable systems; USN; N. Joshi
- Prof Alan Huckleberry; Ruhr-Universitat Bochum Institut Fur Mathematik; 25 November to 21 December 2007; analysis and geometry; ANU; Alexander Isaev
- Jan Saxl; Cambridge University; mid-November to December 2007; -; UWA; Cheryl Praeger
- Ms Weiwei Ren; Yunnan University, China; February to Dececeber 2007; -; UWA; A/Prof Caiheng Li
- Mr Mohamad-Reza Mohebbi; Tehran University of Medical Sciences, Iran; 18 March 2007 to 1 February 2008; -; UMB; -
- Dr Tom Ter Elst; University of Auckland; 20 January to 16 February 2008; analysis and geometry; ANU; Alan McIntosh
- A/Prof Andrea Previtali; University of Insubria-Como; 1 October 2007 to 28 February 2008; computational Group Theory; USN; J.J. Cannon
- Dr M. Iranmanesh; Yazd University, Iran; 10 June 2007 to 10 March 2008; -; UWA; Prof Cheryl Praeger
- Dr Tim Burness; University of Southampton; 21 January to 21 March 2008; -; UWA; Prof Cheryl Praeger
- Prof Philip Griffin; Syracuse University; 01 February to 30 April 2008; Centre for Financial Mathematics; ANU; Alan Welsh
- Dr Youyun Li; Hunan Changsha University; 1 May 2006 to 1 May 2008; -; UWA; A/Prof Song Wang
- Dominic Schuhmacher; University of Zurich; 1 April 2006 to 31 May 2008; -; UWA; Prof Adrian Baddeley
- Dr Ashraf Daneshkhah; Bu-Ali Sina University; 3 September 2007 to June 2008; -; UWA; Prof Cheryl Praeger
- Prof Alireza Ematollahi; Shiraz University; 15 September 2007 to 15 September 2008; -; ANU; Alan Welsh
- Prof David Gubbins; Leeds; 20 September 2007 to 19 September 2008; magneto-hydrodynamic dynamo theory and the geodynamo; USN; D.J. Ivers
- Dr Frederic Robert; Université de Nice; 14 November 2007 to 03 November 2008; applied and nonlinear analysis; ANU; Florica Cirstea
- Dr Alireza Nematollani; University of Shiraz; 15 December 2007 to 15 December 2008; multivariate analysis and time series; USN; N.C. Weber

## The Australian Mathematical Society

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### Membership and Correspondence

Applications for membership, notices of change of address or title or position, members' subscriptions, correspondence related to accounts, correspondence about the distribution of the Society's publications, and orders for back numbers, should be sent to the Treasurer. All other correspondence should be sent to the Secretary. Membership rates and other details can be found at the Society web site: <http://www.austms.org.au>.

### Local Correspondents

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