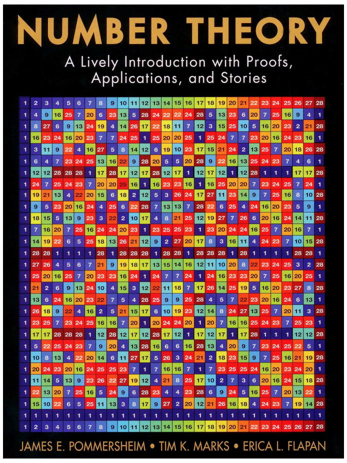
Book reviews

Number Theory

J.E. Pommersheim, T.K. Marks and E.L. Flapan
John Wiley, 2010, ISBN: 978-0-470-42413-1

This is a most unusual textbook on elementary number theory, aptly subtitled *A lively introduction with proofs, applications, and stories*. While it covers roughly the same ground as the classical texts of Niven and Zuckerman's *Introduction to the Theory of Numbers* and LeVeque's *Fundamentals of Number Theory*, as well as more recent results, it is unique in being squarely aimed at the 21st-century student.



By this I mean that the serious bits are covered at a leisurely pace, interspersed with jokes, anecdotes and asides addressed to the student. There are three recurrent cartoon characters: Naomi, a computer wiz, who likes to test theorems numerically; Paul, who restates theorems in plain English after they have been presented in formal mathematical terms; and Phil Lovett (get it?) who makes sometimes plausible but often outlandish conjectures which are later shot down by crucial counter-examples.

Each chapter begins with a short biography of a relevant mathematician, together with a sample of his or her work. Then follows what the authors call a MathMyth, a fantastic story that serves to introduce the main topic in a student-friendly manner. After a systematic presentation of the topic comes a host of exercises, firstly numerical problems and then those requiring reasoning and proofs. Finally there are a number of explorations extending the chapter topic.

I illustrate the approach by considering in detail Chapter 4 on Euclid's algorithm for the greatest common divisor. This begins on p. 148 of a 750-page book, whereas Niven and Zuckerman present it on p. 7 of their 270-page book. Euclid's biography places him in the context of Greek Alexandria and describes his relationship with the emperor Ptolemy, including the usual apocryphal legends. The biography describes his results on right triangles, both real and integral, as well as ruler-and-compass constructions.

The MathMyth introducing the Euclidean algorithm concerns a baker called Euclid in 3rd century BC Alexandria. He is asked to bake a rectangular cake of dimensions 175×65. But Euclid only has square pans, so he fills up the rectangle with the largest possible square cakes, using the greedy algorithm. He manages to do this with two 65×65, one 45×45, two 20×20 and four 5×5 squares. Draw the picture and you will see a 'Proof without Words' that the Euclidian algorithm always terminates with the greatest common divisor of any two given integers.

The actual proof of the Euclidean algorithm is the standard one, but the authors are careful at each step to explain which of the previously defined properties of the integers are used. Naomi explains the precise connection between the geometrical MathMyth and the numerical proof, and Paul puts the calculation of the gcd into prose with a short English sentence.

The Exploration section studies the complexity of the Euclidean algorithm. It contains a proof that the number of steps to compute $\gcd(a, b)$ is less than $\log_2(ab)$ and describes the connection with Fibonacci numbers. This is all nicely illustrated by a table showing each of the 49 steps used to compute the greatest common divisor of two 25-digit integers. Finally, the method is extended to real numbers to illustrate the method of anthyphairesis which will later be employed in the chapter on continued fractions.

The first three chapters cover what I would describe as 'High school number theory', such as the number systems, prime decomposition, induction, modular arithmetic and divisibility. Then comes 'University number theory', starting with the Euclidean algorithm as described above, followed by chapters on Linear Diophantine Equations and the Fundamental Theorem of Arithmetic (i.e. a proof of the uniqueness of prime decomposition). Other topics include Congruence with applications to check-digit schemes and the Gregorian and Mayan calendars, Modular Number Systems, Fermat's Little Theorem and Euler's Theorem with applications to RSA encryption, Primitive Roots, Quadratic Residues, with proofs of quadratic reciprocity, Primality Testing, including both probabilistic and deterministic tests, Gaussian Integers, Continued Fractions and finally Nonlinear Diophantine Equations, including Pell's Equation and some of the more elementary advances towards a proof of Fermat's Last Theorem.

On the whole, I recommend this book enthusiastically as a Number Theory text that students will actually read and enjoy. Unfortunately, it is marred by several typographical errors which may be puzzling to students. Other indications of overhasty production include an intriguing picture on the cover of a 28×28 matrix whose entries are colour-coded integers between 1 and 28. There is no indication in the Preface or Contents pages of the meaning of this picture. A careful examination reveals that it is a table of powers of elements of Z_{29}. Sure enough, one finds an explanation on p. 390 that it is an encryption table for an exponentiation cipher modulo 29. A less forgivable lapse is repeated references to an online Appendix, Student Companion Site and Instructor Companion Site. There is no indication of how these resources can be accessed, nor how the Instructor Site, presumably containing solutions, is made unavailable to students.

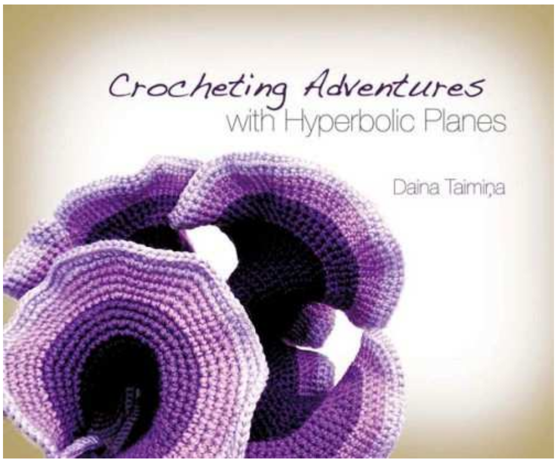
Phill Schultz
School of Mathematics and Statistics, The University of Western Australia, Nedlands, WA 6009.
E-mail: schultz@maths.uwa.edu.au

◇ ◇ ◇ ◇ ◇ ◇

Crocheting Adventures with Hyperbolic Planes

Daina Taimiņa
A K Peters Ltd, 2009, ISBN: 978-1-56881-452-0

The links between mathematics and art have been explored in many works (e.g. [1], [2], [3]). These days, there is a periodical dedicated to these connections, namely the *Journal of Mathematics and the Arts*. In this tradition, *Crocheting Adventures with Hyperbolic Planes* explores the relationship between the art of crocheting and the geometry of surfaces.



How are mathematics and crochet related? Looking back over many years of dressmaking, knitting and crocheting, I find that I am constantly surprised by how many sophisticated mathematical concepts I have used, quite unconsciously. For example, many years ago I used the ideas discussed in this book to construct a costume for a Spanish dancer, when it was important to have very full ruffles on the skirt, but even more important to reduce bulk at the dancer's waistline.

We will describe some chapters in detail in order to convey a sense of the text.

In the Preface to *Geometry and the Imagination* [4], Hilbert states that the aim of the authors is to demonstrate that mathematics is much more than arithmetic by 'offering, instead of formulas, figures that may be looked at and that may be easily supplemented by models which the reader can construct' [4, p. iv]. Similarly, in the introduction to *Crocheting Adventures with Hyperbolic Planes*, Taimiņa states that she is 'sharing a tactile way of exploring mathematical ideas'. Taimiņa is an artist, who has participated in numerous art exhibits, as well as a mathematician. Her work shown in Figure 1 was created for an art exhibit *From Baltic Sea to Coral Reefs*.

Chapter 3 is entitled 'Four strands in the history of geometry'. The author argues that geometry has evolved from (i) art and patterns, (ii) buildings and structures, (iii) navigation, and (iv) motion or machines. The influence of each of these strands is considered in detail and accompanied by many fine illustrations. The author puts Euclid's *Elements* into the second strand. This chapter encourages us to see geometric patterns in unlikely places, and hence, broaden our appreciation of the scope for applying mathematics. Chapter 5 contains a nice introduction to the history of non-Euclidean geometry.

Chapter 4, which is entitled 'Tidbits from the History of Crochet' is just that, tidbits. The author includes much speculation from many sources concerning the origins of crochet, but makes no mention of the extreme popularity of crochet as a means of creating decorative garments during the nineteenth and twentieth centuries. It seems to me that it would be impossible to pinpoint an origin for crochet, because I have seen so many children discover the basic technique simply by playing with a piece of yarn and looping it over their fingers to create a basic crochet chain without a hook. It was called 'finger knitting' when I discovered it, many years ago. I also find it a little odd that images of knitted garments (mittens) are used to illustrate a book on crochet as they do not explore the possibilities of crochet itself.

Figure 1. *The Land and The Sea*, mixed yarn, D. Taimiņa (2009). Photo: Courtesy of the artist.

Chapter 7 is entitled 'Metamorphoses of the hyperbolic plane'. In this chapter the author endeavours to bring together her interests in mathematics and the art of crocheting. To write about mathematics for a general audience — and to write about crocheting for mathematicians — are two challenging tasks. The chapter opens with the author wondering about where mathematicians find their inspiration. Daina Taimiņa finds her inspiration through the interaction between mathematics and her crocheted models. In this chapter, as elsewhere in the book, there are ideas for people with many interests. Primary school teachers will find interesting exercises with paper and cardboard about drawing on surfaces. The photos may suggest that teachers ask students to reflect on the shapes of things around them. Would a mathematics class benefit from visiting the local art gallery? University students may turn to the work of Hilbert and Cohn-Vossen [4] for clarification on aspects of hyperbolic geometry.

The author concentrates exclusively on one crochet stitch, the double crochet stitch (or, in America, the single crochet). It is, of course, the most suitable for her purpose as this stitch forms the basic building block in the crochet repertoire. It enables the construction, not only of these fantastic curves, but also of solid fabrics, very complex lace patterns and elaborate beading, which is one of the most widespread uses for the Tambour technique mentioned on page 64 and one which does not discard the background fabric. The beauty of crochet lies in the range of possible uses for something that is so essentially simple.

This book has had a variety of impacts and connections. It won the 2009 Diagram Prize, awarded by *The Bookseller* magazine, to that year's book with the oddest title. In 2009, the Powerhouse Museum in Sydney was host to *The Sydney Hyperbolic Crochet Coral Reef*, which was part of the the world-wide *Hyperbolic Crochet Coral Reef* project being conducted by *The Institute of Figuring*. The Sydney exhibition brought together mathematics and crochet to raise awareness about the plight of the Great Barrier Reef. At a personal level, the book has drawn our attention to a branch of mathematical activity of which we were previously unaware.

Crocheting Adventures with Hyperbolic Planes demonstrates that abstract mathematical ideas can be sources of inspiration for artists, and that art can be used to demonstrate geometric ideas. The book is richly illustrated with photographs and coloured illustrations and it has been produced on high-quality paper. It would be a useful addition to the library of a school or university. Indeed, this high level of interest shown by artists in geometry may encourage schools and universities to devote more space to geometry in the curriculum.

References

[1] Belcastro, S.-M. and Yackel, C. (eds) (2008). *Making Mathematics with Needlework.* A K Peters Ltd, Wellesley, MA.
[2] Escher, M.C. (c1992). *M.C. Escher, the graphic work : Introduced and explained by the artist.* Germany : Benedikt Taschen.
[3] Gerd Fischer (ed.) (1986). *Mathematical Models from the Collections of Universities and Museums*, Two volumes. Fried. Vieweg & Sohn, Braunschweig.
[4] Hilbert, D. and Cohn-Vossen, S. (1952). *Geometry and the Imagination.* Chelsea, New York.

Frances Mills[1] and Terry Mills[2]
[1]School of Communications, Arts and Critical Enquiry, La Trobe University.
[2]Department of Mathematics and Statistics, La Trobe University, PO Box 199, Bendigo, VIC 3552.

◇ ◇ ◇ ◇ ◇ ◇