Book Reviews

Why is there Philosophy of Mathematics at all?

Ian Hacking is a retired Professor of Philosophy of the Collège de France and the University of Toronto, specialising in the philosophy of science. He is also the author of popular books on logic and the history of probability.

His book is ostensibly addressed to his fellow philosophers, but because of its lively style and provocative title, it is likely to appeal to educated lay readers and especially to mathematicians. Hacking's argument is that mathematics has a special rôle in philosophy because it exemplifies the fundamental philosophical concepts of existence (ontology) and knowledge (epistemology) as well as logic.

For example, Pascal, Descartes and Leibniz are equally renowned as philosophers and mathematicians; many philosophers such as Bacon, Berkeley, Kant, Frege, Bernays, Lakatos, Kuhn, Pierce, Quine, Russell and Wittgenstein continually referred to mathematics; and mathematicians like Cantor, Dedekind, Gödel, Hamilton, Hardy, Hilbert, Kronecker, Littlewood, Weyl and Weil contributed to philosophical discourse. Hacking also describes recent forays by Atiyah, Connes, Gowers, Grothendieck, Selberg, Thurston, and Voevodsky into philosophical argument. While Hacking's work is strong on the Western historical background, he ignores contributions of the medieval Islamic philosophers.
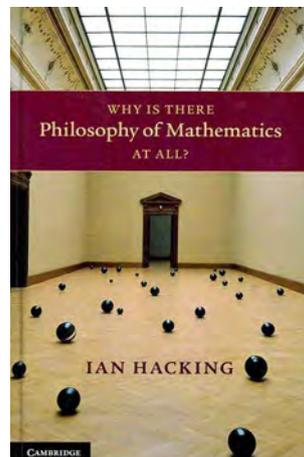
What precisely is the philosophical argument, and how does it impact on mathematics? The two major opposing schools in analytic philosophy today are *realism* and *nominalism*. Hacking's amusing example discusses the two statements: 'Jupiter has 67 moons' and '67 is the number of moons of Jupiter'. They seem to assert equivalent propositions but they mean different things to realists and nominalists.

As far as mathematics is concerned, realism, which includes platonism and intuitionism, asserts that mathematical objects and structures exist and the mathematician's job is to discover them, whereas nominalism, associated with constructivism and formalism, asserts that mathematics is invented by human minds deducing conclusions by explicit rules from more or less arbitrary hypotheses. But whereas philosophers are usually in one or the other camp and boldly declare their allegiance, mathematicians are more circumspect and willingly switch when they deem it appropriate. For example, who could deny that Group Theory is invented,

but the sporadic simple groups discovered? On the other hand, the independence of the continuum hypothesis seems to put paid to outright platonism. Hacking himself fairly and forcefully presents both sides of the mathematical argument but does not embrace a single alternative.

One of the major themes of the book is proof. How did it arise in mathematics, how has it changed through history, and where is it headed? A novel viewpoint is Hacking's classification of mathematical proof as either Leibnizian or Cartesian. The first denotes a proof which consists of a finite sequence of statements each of which is an axiom or follows from previous statements in the list by *modens ponens* or some other rule of inference. This definition can be modified to allow recursively defined infinite natural deduction trees. On the other hand, a Cartesian proof is one which you can and must grasp as a whole. Examples, other than those in Descartes' *Meditations*, include Plato's proof of Meno's slave that you double the area of a square by constructing the square on the diagonal, and Littlewood's proof that while a square can be dissected into smaller squares each of a different size, a cube cannot be dissected into smaller cubes each of a different size.

Incidentally, this is one part of the book where I disagree with Hacking. He claims that proof began with Thales and is unique to Western mathematics. This may be true if you limit yourself to Leibnizian proof, but many examples of Cartesian proof can be found in Arabic, Chinese and especially Indian mathematics.

Another recurrent theme in the book is the relation between pure and applied mathematics. Hacking points out that well before any distinction was made between them, there was 'mixed' mathematics, even by that name, in Francis Bacon's work. It meant simply that one can apply Euclidean geometry to ballistics or mechanics, for example, by adjoining an empirical component. The same concept occurs in Galileo's work on sliding and freely falling bodies. Other applications discussed include Newtonian physics, probability, the 19th century division of mathematics into pure and applied and its effect on mathematical education, and the philosophical implications of the continuing division.

Hacking has read broadly in areas of current mathematical interest, such as the Langlands program and univalent foundations, but admits his ignorance of the details. Therefore it would be unfair to dwell on the mathematical errors in this book, such as his belief that there are 26 finite simple groups.

His message is that mathematics is of continuing interest and importance to analytic philosophy. Whether philosophy has anything to offer mathematics is a matter of opinion. Many noted mathematicians believe it has; my personal view is that it is a waste of time to worry about whether 67 or even the set of whole numbers exist, but it is worthwhile occasionally to step back from the coalface and consider the meaning and importance of our work.

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