Symbols: do university students mean what they write and write what they mean?

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MATHEMATICS
Do they speak the same language ?

Overview

- Symbolic literacy
  - what is it?
  - why is it important?

- Frameworks for the study of symbolic literacy

- Application of frameworks:
  - pilot study
  - some illustrations

- Implications

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Symbolic Literacy

- Symbols: basis of mathematical language
- Symbolic literacy: read & write
- Symbol sense

Symbolic literacy: *near enough is not good enough*

Common symbolic statements may look very similar but have quite different meanings and results:

$$(-1)^2 \qquad -1^2$$

$$[2+6\ \times\sqrt{4}]^2 \qquad [(2+6)\ \times\sqrt{4}]^2$$

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Basis for framework

Following Serfati we consider :

- *Materiality*

  ‘physical’ attributes (what it looks like)

  category (a letter, a numeral, a specific shape, etc.)

- *Syntax*

  the ‘rules’ to be considered when writing a symbol

- *Meaning*

  the concept being conveyed

  as commonly agreed by the community of mathematicians

Basis for framework

Sherin:

Close to Serfati's concept of 'syntax':

- the *symbol template* *or* ***syntax template*** specifies how that idea is written in symbols

Pilot study: data collection

- Participants

    tutorial class 1st year uni maths students (21)

    convenience sample

- Context

    first semester Calculus 1 tutorial class

    Students stand and work on wall mounted white boards – usually in pairs

    Tutor 'roves'

- Data collection

    Observation

    Photographs of student work

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Pilot study: data analysis

Trialled used of framework for symbolic literacy by analysing incorrect solutions.

Serfati's (2005) notions of **materiality**, **syntax** and **meaning** and incorporating Sherin's (1996) idea of symbol template (here '**syntax template**').

General comments on the data

- For most of these students, the week of tutorial 7, which had included two lectures on the topic, was their first encounter with complex numbers.
- The **materiality**, that is, the 'shapes' of the symbols and their combination with other symbols, were all **familiar** from school algebra but some of the **syntax and meaning** were **not**.

  students were familiar with latin letters standing for unknowns, variables, etc., the letter *i* in a complex number takes a very precise and new meaning

  students were familiar with square roots applicable to positive numbers, here the syntax of 'square root' is expanded to include negative numbers

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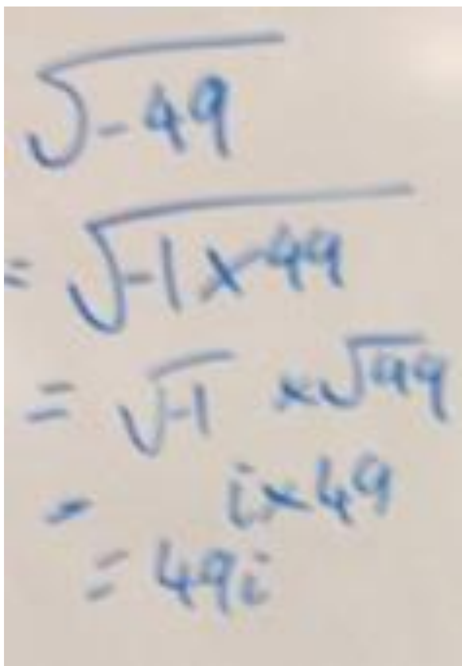
General comments on the data

It was clear that every example in these practice exercises involved complex numbers so students were focusing on applying their new learning. In these circumstances it seems that **errors in their established templates for syntax** are exposed.

Illustration 1 (writing)

Simplify $\sqrt{-49}$ expressing your answer in Cartesian form *a* + *ib* where *a* and *b* are real numbers.



'Omitted' to take the square root of 49. Yet not a mere case of 'having forgotten'. Potential source: the difference in meanings of a same materiality of symbol $\sqrt{\ }$

- previously decoding $\sqrt{\ }$ as a process (take the square root of), now $\sqrt{-1}$ must be considered as a 'block' instead of symbolic template $\sqrt{\blacksquare}$ **OK (line 3 –>line4) but**
- **At the same time** sees $\sqrt{-1}\times\sqrt{49}$ with syntax template $\sqrt{\blacksquare}\times\sqrt{\blacksquare}$ and (wrongly) applies the properties for square roots

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Illustration 2 (reading)

Being **symbolic literate** also means, in some sense, to appropriately read and make meaning of what is asked, including having to sometimes **decode 'hidden messages' in the stimulus**.

Consider:

Find the modulus of the following complex number without multiplying into Cartesian form:

$$\frac{-5i(3-7i)(2+3i)}{(6+4i)(7+3i)}$$

Illustration 2

$$\frac{-5i(3-7i)(2+3i)}{(6+4i)(7+3i)}$$

- Students must ‘lock’ the meaning of *i* as a symbol standing for the imaginary unit, *without further considering its intrinsic property*.

- If the students replaced *i* by $\sqrt{-1}$, that would lead them to the numerical dead end

$$\frac{-135\sqrt{-1}-25}{46\sqrt{-1}+30}.$$

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Illustration 2

$$\frac{-5i(3-7i)(2+3i)}{(6+4i)(7+3i)}$$

- Note prompt : “without multiplying into Cartesian form” so 3–7i should now **to be seen as a whole**
- i has the same syntax as any other letter so the temptation is to apply the distributive law to
  (3–7i)(2+3i). (eventually leading to 5/2)

Illustration 2

- Underlying the question is the need to work with properties of modulus of complex numbers (modulus of product of complex numbers).

- Not apply algebraic manipulations as one would for syntactically similar expressions.

- Go beyond syntax template $\square - \square i$ and

- view it as a complex number.

- The **context signals** an efficient approach to finding the appropriate answer.

Illustration 2

Find the modulus of the following complex number without multiplying into Cartesian form:

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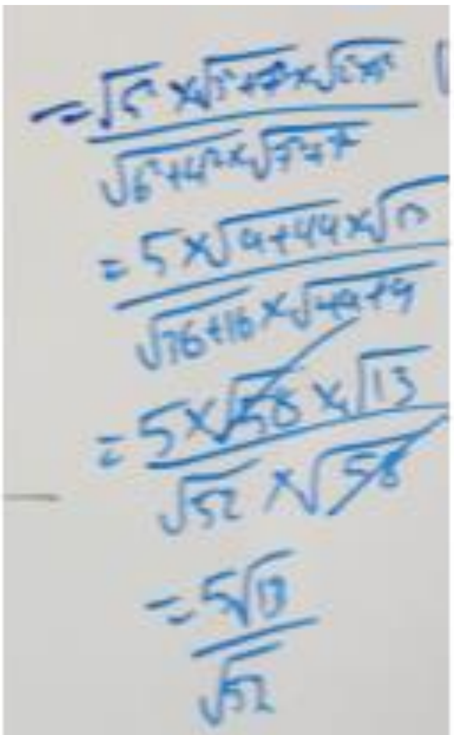

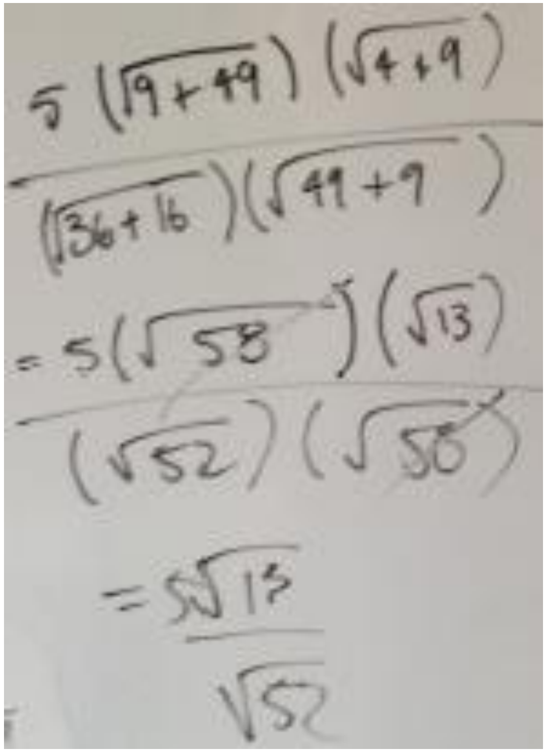

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Illustration 2

Find the modulus of the following complex numbers without multiplying into Cartesian form:

$$\frac{-5i(3-7i)(2+3i)}{(6+4i)(7+3i)}$$

- Students recognised each element of the expression as a given complex number
- They correctly apply the definition of modulus of complex numbers and their properties.
- They carry out correct mathematical procedures to finally provide numerical answers.

BUT

Illustration 2

$$\frac{-5i(3-7i)(2+3i)}{(6+4i)(7+3i)}$$

- A more efficient solution to the problem requires interpreting the modulus of complex numbers without necessarily having recourse to the Pythagorean formula, and to rather interpret the meaning of e.g. $|3-7i|$ (and all other expressions) in the geometrical sense.

- Having done so, students would have been able to 'cancel out' pairs of modulus (e.g. $|3-7i|$ and $|7+3i|$) and come up with a very much more efficient solution.

complexity of being able to navigate between meanings of expressions 'with same materiality'

Implications (1)

- Consideration of materiality is important for both teachers and students
- The choice of letters and form of the symbol act as a cue to the student in making choices about efficient solution methods (Illustration 2)
- Teachers need to help their students learn to recognise such cues
- Students need to take a moment to consider the makeup of each symbol rather than relying on unthinking recognition of syntax templates

Implications (2)

The notion of syntax templates can help teachers identify likely causes for students' errors and provides a way of talking about the structure and meaning of symbols where in one context students need to recognise a symbol as indicating a process but in another identifying a combination signifying a concept (Illustration 1)(Tall et al. 2001).

Implications (3)

- Students should expect mathematics to be read with logical meaning
- Mathematical literacy (Usiskin 2012) may be promoted through contemplation of syntax templates by both teachers and students.

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Questions

- Reactions? Questions?

- Follow up

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